MAE502 2011 Homework #5

Prob 1 (3 points)
For $u(x,t)$ defined on $0 \le x \le 1$ and $t \ge 0$, solve the nonhomogeneous PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \cos(\pi x)e^{-2t} + \cos(3\pi x)e^{-t} \quad ,$$

with boundary conditions

(I) $u_x(0, t) = 0$ (II) $u_x(1, t) = 0$ (u_x is $\partial u/\partial x$)
(III) $u(x,0) = 5\cos(2\pi x) + 2\cos(3\pi x)$.

Use the solution to evaluate $u(x,t)$ at $(x,t) = (0.6, 0.1)$ and $(0.6, 0.3)$.

Prob 2 (4 points)
(a) For $u(x,t)$ defined on the infinite domain, $-\infty < x < \infty$, and $t \ge 0$, use Fourier transform method to solve the nonhomogeneous PDE

$$\frac{\partial u}{\partial t} = -0.05\frac{\partial^4 u}{\partial x^4} + q(x,t) \quad ,$$

with boundary conditions

(I) $u(x, t)$ and all its partial derivatives in x vanish as $x \to \infty$ and $x \to -\infty$.
(II) $u(x,0) = \mathrm{P}(x)$,

where

$q(x,t) \equiv \mathrm{R}(x)\mathrm{S}(t)$,

$\mathrm{R}(x) = -1$ if $-2 \le x \le -1$
$= 0$ otherwise ,

$\mathrm{S}(t) = 1$ if $t \le 1$
$= 0$ if $t > 1$,

and
$\mathrm{P}(x) = 1$ if $1 \le x \le 2$
$= 0$ otherwise

(b) Evaluate and plot your solution as a function of x for $t = 0.3, 0.6, 1.0, 1.2$, and 1.5. (This part is important and will account for at least 50% of the score.)