MAE502 2011 Homework #6

Prob 1 (3 points)
Solve the PDE for $u(x,t)$ defined on the infinite domain, $-\infty < x < \infty$, and $t \geq 0$,

$$\frac{\partial u}{\partial t} + t\,u\frac{\partial u}{\partial x} = 0$$

with the boundary condition

$u(x,0) = \mathrm{P}(x)$,
where

$$\begin{aligned} \mathrm{P}(x) &= 1 \quad , \text{ if } x < 0 \\ &= 1 + x \quad , \text{ if } 0 \leq x \leq 1 \\ &= 2 \quad , \text{ if } x > 1 \ . \end{aligned}$$

Plot the solution as a function of x for $t = 1$ and $t = 2$, along with the initial state ($t = 0$). Sketch the characteristics in the x-t plane.

Prob 2 (4 points)
Solve the PDE for $u(x,t)$ defined on the infinite domain, $-\infty < x < \infty$, and $t \geq 0$,

$$\frac{\partial u}{\partial t} + 3\,u\frac{\partial u}{\partial x} = t$$

with the boundary condition

$u(x,0) = \mathrm{P}(x)$,
where

$$\begin{aligned} \mathrm{P}(x) &= 1 \quad , \text{ if } x < 0 \\ &= 2 \quad , \text{ if } x > 0 \ . \end{aligned}$$

See diagram below. (At $x = 0$, $\mathrm{P}(x)$ has all the values between 1 and 2; see a relevant example in pp. 565-566) Plot the solution as a function of x for $t = 1$ and $t = 2$, along with the initial state ($t = 0$). Sketch the characteristics in the x-t plane. What are the values of $u(x,t)$ at ($x = 27$, $t = 3$) and ($x = 11$, $t = 1.5$)?