MAE/MSE 502 Spring 2012 Homework #5

Prob 1 (4 points)

For $u(x, t)$ defined on the domain of $0 \le x \le 1$ and $t \ge 0$, solve the nonhomogeneous PDE,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - 0.5u - t + \cos(\pi x)\exp(-2t)$$

with the boundary conditions:

$u_x(0, t) = 0$ (u_x is $\partial u/\partial x$)
$u_x(1, t) = 0$
$u(x, 0) = 2 + \cos(\pi x) + 0.5\cos(3\pi x)$

Evaluate and plot $u(x, t)$ as a function of x at $t = 0$ (initial state), 0.1, 0.2, 0.5, and 1.

Prob 2 (4 points)

For $u(x, t)$ defined on the infinite domain of $-\infty < x < \infty$ and $t \ge 0$, solve the nonhomogeneous PDE,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + Q(x, t) \quad ,$$

with the boundary conditions:

(i) u and u_x vanish as $x \to \infty$ and $x \to -\infty$
(ii) $u(x,0) = P(x)$,

where

$$Q(x, t) \equiv S(x)\exp(-t) \ ,$$

$S(x) = 0.5$, if $-1 \le x \le 1$
$= 0$, otherwise ,

and

$P(x) = 1$, if $2 \le x \le 3$
$= 0$, otherwise

Plot $u(x, t)$ as a function of x at $t = 0.2$ and 1, along with the initial state, $u(x, 0)$.