MAE/MSE502 Spring 2012 Homework #6A
Deadline for this problem is the same as HW6

Problem 1 (3 points)
For $u(x, y)$ defined on the square domain of $0 \le x \le 1$ and $0 \le y \le 1$, use one of the numerical methods in Chapter 6 (see instruction below) to solve the Laplace's equation taken from HW2 Prob 2,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad ,$$

with the boundary conditions,

(i) $u(0, y) = 0$
(ii) $u(1, y) = \sin(\pi y) + 0.5 \sin(3\pi y)$
(iii) $u(x, 0) = 6\,(x - x^2)$
(iv) $u(x,1) = 0$.

Find the numerical solutions for the two cases with ($\Delta x = 0.05$, $\Delta y = 0.05$) and ($\Delta x = 0.2$, $\Delta y = 0.2$) where Δx and Δy are the grid sizes in the x and y directions. Make a color/contour plot of the solution for each case. Find the value of $u(x, y)$ at ($x = 0.6$, $y = 0.3$) for the two cases and compare them with the analytic solution from HW2 Prob 2.

General suggestion:
To proceed, discretize the 2nd partial derivatives in both x and y directions by using the 2nd order centered finite difference scheme (e.g., **Eqs. 6.2.15-6.2.17** in textbook). This will turn the PDE into a standard system of linear equations,

$$[\mathbf{A}]\, \mathbf{u} = \mathbf{b} \ ,$$

where **[A]** is a matrix, **u** is a vector that contains the values of $u(x,y)$ at the discrete grid points. (It might be useful to consolidate the two indices in the x and y directions into a single index for the **u** vector.) Since the system is not very large, you may be able to directly invert it, i.e., by expressing the solution as

$$\mathbf{u} = [\mathbf{A}]^{-1}\, \mathbf{b} \ .$$

This can be done using typical matlab functions for matrix manipulations. (As usual, you do not have to use matlab if you prefer to use other softwares of your choice.) Alternatively, you may find the iterative methods in **Sec. 6.6** useful for solving the discretized system. In that case, the Gauss-Seidel method is a good choice. Be careful in inserting the boundary conditions into the discretized equations. We will discuss this point in class.