MAE/MSE502 2012 Homework #6B
Please return the solutions for HW6A and HW6B together

Prob 1 (3 points)
For $u(x,t)$ defined on the infinite domain, $-\infty < x < \infty$, and $t \geq 0$, solve the PDE,

$$\frac{\partial u}{\partial t} + t\,u\frac{\partial u}{\partial x} = -3\,u \quad ,$$

with the boundary condition

$$u(x,0) = \exp(-x^2) \ .$$

Plot the solution as a function of x for $t = 0.2$ and 0.5, along with the initial state ($t = 0$). Sketch the characteristics in the x-t plane.

Prob 2 (4 points)
For $u(x,t)$ defined on the infinite domain, $-\infty < x < \infty$, and $t \geq 0$, solve the PDE,

$$\frac{\partial u}{\partial t} + (u+1)\frac{\partial u}{\partial x} = 0 \quad ,$$

with the boundary condition

$$u(x,0) = \mathrm{P}(x),$$

where

$$\begin{aligned} \mathrm{P}(x) &= 1 \quad &&, \text{ if } x \leq 0 \\ &= 1 + x^2 &&, \text{ if } 0 < x \leq 1 \\ &= 2 &&, \text{ if } x > 1 \ . \end{aligned}$$

Plot the solution as a function of x for $t = 1$ and 2, along with the initial state ($t = 0$). Sketch the characteristics in the x-t plane. What are the values of $u(x,t)$ at ($x = 4$, $t = 1.5$) and ($x = 9$, $t = 2.5$)?

Prob 3 (1 point)
Try to convert the following second-order PDE,

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial t} + 4\frac{\partial u}{\partial x} - 3\,u = 0 \quad ,$$

into a set of first-order PDEs. There might be more than one solutions for this problem. You only need to provide one example.