MAE502 Spring 2013 Homework #2

Prob 1 (2 points)
Solve the 1-D wave equation for $u(x, t)$ defined on the domain of $0 \le x \le 1$ and $t \ge 0$,

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \quad ,$$

with the boundary conditions,

(I) $u(0, t) = 0$
(II) $u(1, t) = 0$
(III) $u(x,0) = x - x^2 + 0.1\, sin(2\pi\, x)$
(IV) $u_t(x,0) = 0$ (u_t is $\partial u/\partial t$.)

Plot the solution, $u(x, t)$, as a function of x at $t = 0$, 0.3, 0.5, 0.7, 1.0, and 1.6.

Prob 2 (2 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 3$ and $t \ge 0$, consider the modified one-dimensional heat equation,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 50u \quad ,$$

with the boundary conditions,

(I) $u(0, t) = 0$, (II) $u(3, t) = 0$, and (III) $u(x,0) = P(x)$.

(a) Find the solution for the case with $P(x) \equiv \sin(4\pi x)$. Discuss the behavior of the solution as $t \to \infty$.
(b) Find the solution for the case with $P(x) \equiv \sin(2\pi x) + 0.5\sin(5\pi x)$. Discuss the behavior of the solution as $t \to \infty$.
(Hint: This is still a homogeneous PDE. The usual procedure of separation of variables and eigenfunction expansion should work.)

Prob 3 (3 points)
Solve the 2-D Laplace's equation defined on the domain of $0 \le x \le 1$ and $0 \le y \le 1$,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad ,$$

with the boundary conditions,

(I) $u(0, y) = 0$
(II) $u(1, y) = y - y^2 + 0.1\, sin(2\pi\, y)$
(III) $u(x, 0) = 0$
(IV) $u(x, 1) = sin(\pi\, x)$.

Make a contour/colorfill plot of the solution, $u(x,y)$. The recommended function to use in Matlab is `contour` or `contourf`. What is the value of $u(x,y)$ at $x = 0.6$, $y = 0.4$?