MAE/MSE 502 Spring 2013 Homework #4

Prob 1 (4.5 points) For $u(x, t)$ defined on the domain of $0 \le x \le 1$ and $t \ge 0$, solve the 1-D heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \ ,$$

with the boundary conditions:

(i) $u(0, t) = -2\, u_x(0, t)$ $\quad (u_x \equiv \partial u/\partial x)$
(ii) $u(1, t) = 3$
(iii) $u(x,0) = 3\, x^2$

Plot the solution, $u(x,t)$, as a function of x at $t = 0.2$, 0.5, 0.9, and 1.5, along with the initial state $u(x,0)$ and equilibrium solution $u(x, \infty)$. Please collect all six curves in one plot. Specific to this problem only, to receive full credit you must use at least five eigenfunctions to do the eigenfunction expansion. You must also provide additional information as listed in the following:

(a) Your matlab code (or its equivalent if you use a different software)
(b) The values of the first five eigenvalues (with the smallest absolute value) for the eigenvalue problem in the x-direction.
(c) A plot of the eigenfunctions in the x-direction corresponding to the five eigenvalues you provided in (b). This would form the minimum set of eigenfunctions for you to use in the eigenfunction expansion. Please collect all five curves in a single plot. Although it is not absolutely necessary, it is recommended that the eigenfunctions be normalized for this plot. If $G_n(x)$ is the n-th normalized eigenfunction, it should satisfy

$$\int_0^1 [G_n(x)]^2\, dx = 1 \ .$$

(Hint: You might find the material in Sec 5.8 useful for dealing with some technical aspects of this problem.)

Prob. 2 (3.5 points) For $u(x, y, t)$ defined on the domain of $0 \le x \le 1$, $0 \le y \le 1$, and $t \ge 0$, solve the 2-D wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \ ,$$

with the following boundary conditions:

(i) $u(0, y, t) = 0$
(ii) $u(1, y, t) = 0$
(iii) $u(x, 0, t) = 0$
(iv) $u(x, 1, t) = 0$
(v) $u(x, y, 0) = (x - x^2)(y - y^2 + 0.2 \sin(2\pi y))$
(vi) $u_t(x, y, 0) = 0$ $\quad (u_t \equiv \partial u/\partial t)$.

Plot the solution $u(x, y, t)$ at $t = 0$ (initial state), 0.3, 0.7, and 2.46 as contour or colorfill maps.