MAE 502 Spring 2013 Homework #5

Prob. 1 (4 points)

For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, use the Fourier transform method to solve the PDE

$$\frac{\partial u}{\partial t} = -0.02\frac{\partial^4 u}{\partial x^4} + 0.0005\,\frac{\partial^6 u}{\partial x^6} \quad ,$$

with the boundary condition,

$u(x,0) = \mathrm{P}(x)$,

where

$$\begin{aligned} \mathrm{P}(x) &= 1, \quad -3 \leq x \leq -1 \\ &= 2\,, \quad 1 < x \leq 2 \\ &= 0\,, \quad \text{otherwise.} \end{aligned}$$

Evaluate and plot $u(x, t)$ as a function of x at $t = 0.03$, 0.1, and 0.3. As a reference, please also plot the initial state $u(x,0) = \mathrm{P}(x)$ and collect all 4 curves in one figure.

Note: Numerical integration (e.g., by the trapezoidal method) may be needed to evaluate u(x, t). Since numerical integration cannot go all the way to ∞, one has to "truncate" the integral at a finite value of the "wavenumber" ω. It is part of your job to determine where to make the truncation. A useful way to determine the cutoff is to plot U(ω, 0) (the Fourier transform of u(x, 0)) as a function of ω and observe how it decays with ω.

Prob 2 (2 points)

For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, use the Fourier transform method to find the analytic solution of the PDE,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad ,$$

with the boundary condition

$u(x,0) = \exp(-x^2/4)$.

Hint: You might find the following formula useful:

$$\int_0^\infty e^{-x^2}\cos(2bx)\,dx = \frac{\sqrt{\pi}}{2}e^{-b^2}$$

Prob 3 (3 points)

For $u(x, t)$ defined on the domain of $0 \leq x \leq 1$ and $t \geq 0$, solve the nonhomogeneous PDE,

$$\frac{\partial u}{\partial t} = 0.01\frac{\partial^2 u}{\partial x^2} + u - \sin(t) + t\cos(3\pi x)$$

with the boundary conditions:

$u_x(0, t) = 0$ (u_x is $\partial u/\partial x$)
$u_x(1, t) = 0$
$u(x, 0) = 3 + 2\cos(3\pi x) + \cos(4\pi x)$.

Evaluate $u(x, t)$ at $x = 0.2$, $t = 0.5$.