MAE/MSE 502, Fall 2014, HW3 Solutions

Prob 1

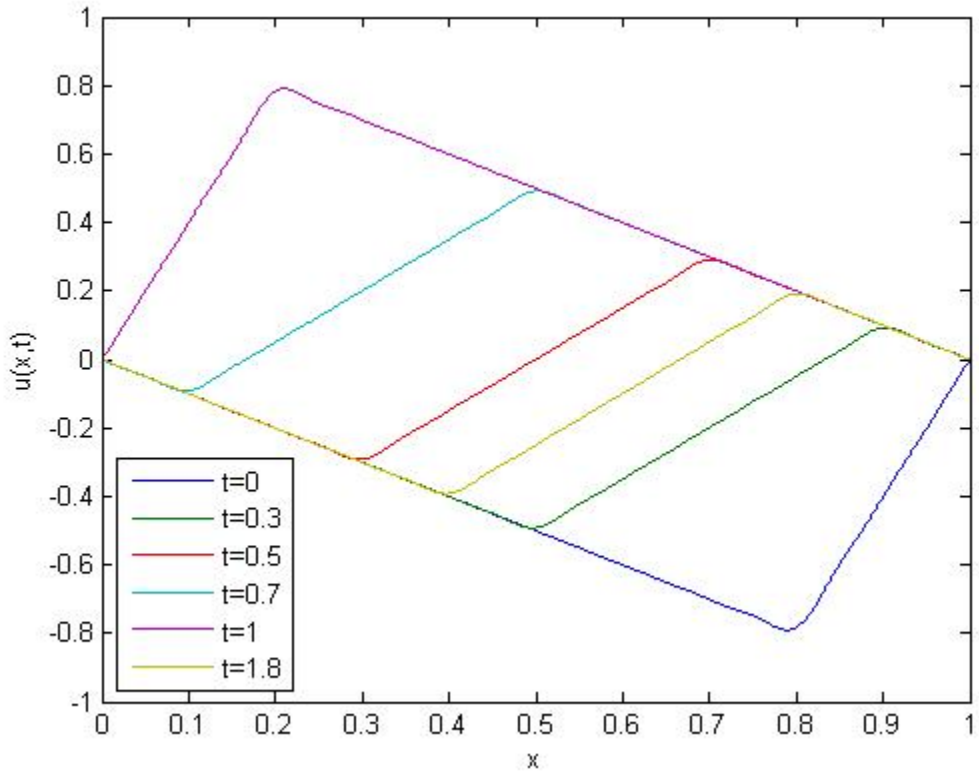
The solution is

$$u(x,t) = \sum_{n=1}^{\infty} a_n \sin(n\pi x)\cos(n\pi t)$$

where

$$a_n = \frac{\int_0^{0.8} -x \sin(n\pi x)dx + \int_{0.8}^{1} 4(x-1) \sin(n\pi x)dx}{\int_0^1 [\sin(n\pi x)]^2 dx} \quad .$$

Plot:



Prob 2

(a) Setting $u(x,y) = G(x)H(t)$, the outcome of separation of variables are

$$\frac{dH}{dt} = c\,H \quad ,$$

and

$$\frac{d}{dx}[(0.5\exp(x^2))\frac{d\,G}{d\,x}] + x^2\,G = c\,G \quad , \; G(2) = 3\,G'(2) \; , G(5) = 0 \; .$$

The system in the x-direction satisfies the standard Sturm-Liouville form.

(b) The solution can be written as

$$u(x,t) = \sum_{n=1}^{\infty} a_n\,G_n(x)\exp(c_n t) \quad ,$$

where

$$a_n = \frac{\int_2^5 P(x)\,G_n(x)\,dx}{\int_2^5 [G_n(x)]^2\,dx} \quad .$$