MAE/MSE 502, Fall 2014 Homework #5

Must submit the printout of Matlab codes for Prob 1 and 2 to receive credit for the plots.

Prob 1 (3 points)

For $u(x,t)$ defined on the infinite domain of $-\infty < x < \infty$ and $t \geq 0$, use the Fourier transform method to solve the PDE,

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} \quad ,$$

with the boundary conditions

(I) $u(x, t)$ and its 1st and 2nd partial derivatives in x vanish as $x \rightarrow \infty$ and $x \rightarrow -\infty$.
(II) $u(x,0) = \exp(-x^2/4)$.

Plot $u(x, t)$ as a function of x at $t = 0$, 1, and 5. Please make the plot over the range of $-20 \leq x \leq 20$.

Prob 2 (3 points)

For $u(x,t)$ defined on the infinite domain of $-\infty < x < \infty$ and $t \geq 0$, use the Fourier transform method to solve the PDE,

$$\frac{\partial u}{\partial t} = -\frac{\partial^4 u}{\partial x^4} + \exp\left(-x^2 - t\right) \quad ,$$

with the boundary conditions

(I) $u(x, t)$ and its 1st-3rd partial derivatives in x vanish as $x \rightarrow \infty$ and $x \rightarrow -\infty$.
(II) $u(x,0) = \exp(-x^2/4)$.

Plot $u(x, t)$ as a function of x at $t = 0$, 1.5, 5 and 10. Please make the plot over the range of $-10 \leq x \leq 10$.

Prob 3 (2 points)

For $u(x,t)$ defined on the infinite domain of $-\infty < x < \infty$ and $t \geq 0$, use the Fourier transform method to solve the PDE,

$$\frac{\partial u}{\partial t} = 3\frac{\partial u}{\partial x} + u \quad ,$$

with the boundary conditions

(I) $u(x, t)$ vanishes as $x \rightarrow \infty$ and $x \rightarrow -\infty$.
(II) $u(x,0) = \exp(-x^2/4)$

We expect a closed-form solution which contains no unevaluated integral. A deduction will be assessed on any unevaluated integral, even if it (when evaluated) will lead to the correct answer. Otherwise, no need to make any plot for this problem.

Note for Prob 1 and 2: Numerical integration (e.g., by the trapezoidal method) will be needed to evaluate $u(x, t)$ for the plots. Since numerical integration cannot go all the way to ∞, one has to "truncate" the integral at a finite value of ω. This is analogous to truncating a Fourier series at a finite n. A useful way to determine where to truncate the integral is to plot, for a give t, $U(\omega, t)$ (the Fourier transform of $u(x, t)$) as a function of ω and observe how $U(\omega, t)$ decays with ω.

Note for Prob 1-3: The following formula is useful for solving the three problems that involve Fourier transform. Specifically, it will help simplify the Fourier transform (but not the inverse transform) in Prob 1 and 2. For Prob 3, applying the formula once in the forward and once in the inverse Fourier transform will lead to a closed-form solution that does not contain any unevaluated integral.

$$\int_0^\infty e^{-x^2} \cos(2bx)\,dx = \frac{\sqrt{\pi}}{2} e^{-b^2} \,.$$

Prob 4. (3 points)

For $u(x,t)$ defined on the domain of $0 \le x \le 1$ and $t \ge 0$, solve the PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 5 + t + \cos(3\pi x)\exp(-t) \quad ,$$

with the boundary conditions

(I) $u_x(0,t) = 0$ (u_x is $\partial u/\partial x$)
(II) $u_x(1,t) = 0$
(III) $u(x,0) = 2 + \cos(\pi x) + \cos(3\pi x)$

We expect a closed-form solution without any unevaluated integral.