MAE/MSE 502, Fall 2014 Homework #6

Prob 1 (3 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, find the solution of the PDE,

$$\frac{\partial u}{\partial t} + (0.5 + u)\frac{\partial u}{\partial x} = 0 \quad ,$$

with the boundary condition,

$u(x, 0) = \mathrm{P}(x)$,

where

$$\begin{aligned} \mathrm{P}(x) &= 1 \quad &&, \text{ if } x \leq 0 \\ &= 1 + x^2 &&, \text{ if } 0 < x \leq 1 \\ &= 2 &&, \text{ if } x > 1 \end{aligned}$$

Plot the solution, $u(x,t)$, as a function of x at $t = 0$, 1, and 2.

Prob 2 (2 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, find the solution of the PDE,

$$0.5\,\frac{\partial u}{\partial t} + x\left(\frac{\partial u}{\partial x} + 1\right) = 0 \quad ,$$

with the boundary condition,

$u(x, 0) = \mathrm{P}(x)$,

where

$$\begin{aligned} \mathrm{P}(x) &= 1 \quad &&, \text{ if } x \leq 0 \\ &= e^{-x} &&, \text{ if } x > 0 \end{aligned}$$

(See Fig. 1 for a plot of P(x).)

Using your solution, evaluate $u(x,t)$ at ($x = 1$, $t = 0.1$) and ($x = -1$ $t = 0.2$).

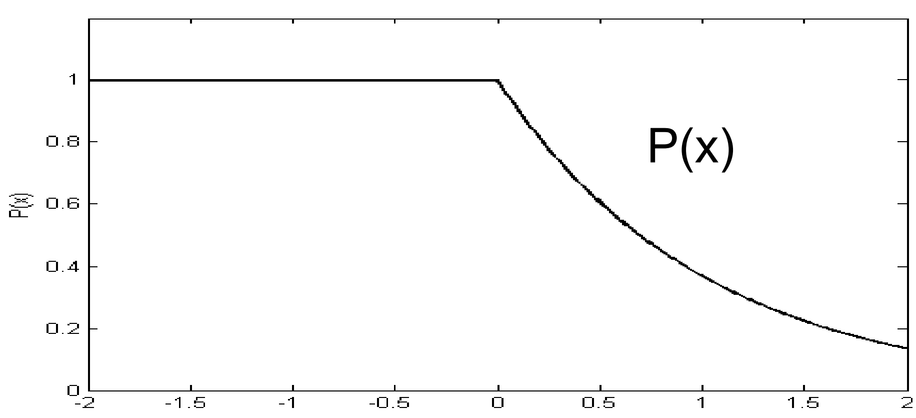

Fig. 1

Prob 3 (2 points)
Consider the following PDE for $u(x, t)$ defined on the infinite domain of $-\infty < x < \infty$ and $t \geq 0$,

$$\frac{\partial u}{\partial t} = 5u + Q(t) \quad ,$$

with the boundary condition,

$$u(x, 0) = \mathrm{P}(x).$$

(a) Find the Green's function, $G(t, t')$, such that for any given $Q(t)$ and $\mathrm{P}(x)$ the solution of the system can be expressed as

$$u(x,t) = G(t,0)P(x) + \int_0^t G(t,t')Q(t')dt' \quad . \qquad \text{Eq. (1)}$$

(b) Use the Green's function from (a) and Eq. (1) to construct the solution for the case with $\mathrm{P}(x) = \exp(-x^2)$ and $\mathrm{Q}(t) = \exp(-3t)$.