MAE502 Spring 2014 Homework #4A

Problem 1 (3.5 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 1$ and $t \ge 0$, solve the Heat equation with an internal heat source/sink, $Q(x)$,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + Q(x) \quad ,$$

with the boundary conditions,

(i) $u(0, t) = 0$, (ii) $u(1, t) = 0$, and (iii) $u(x,0) = P(x)$,

where

$$P(x) = \begin{cases} 0 & , \text{ if } 0 \le x \le 0.25 \\ 4x - 1 & , \text{ if } 0.25 < x \le 0.5 \\ 3 - 4x & , \text{ if } 0.5 < x \le 0.75 \\ 0 & , \text{ if } 0.75 < x \le 1 \end{cases}$$

and

$$Q(x) = \begin{cases} 0 & , \text{ if } 0 \le x \le 0.5 \\ 20 - 40x & , \text{ if } 0.5 < x \le 0.75 \\ 40x - 40 & , \text{ if } 0.75 < x \le 1 \end{cases}$$

Plot the solution as a function of x at $t = 0$, 0.01, 0.05, and 0.2, along with the steady state ($u(x,t)$ as $t \to \infty$). Collect all five curves in a single plot.

Problem 2 (1.5 point)
For $u(x,t)$ defined on the domain of $0 \le x \le 1$ and $t \ge 0$, find the analytic solution of the nonhomogeneous PDE,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - 5\sin(t) + \cos(2\pi x)\exp(-t) \quad ,$$

with the boundary conditions,

(i) $u(0, t) = u(1, t)$
(ii) $u_x(0, t) = u_x(1, t)$
(iii) $u(x, 0) = 4 + 3\sin(6\pi x)$.

Using your solution, evaluate $u(x, t)$ at $x = 0.8$, $t = 0.05$.