MAE/MSE502 Spring 2014 Homework #6A

Prob 1 (3 points)
For $u(x,t)$ defined on the infinite domain, $-\infty < x < \infty$, and $t \geq 0$, use the Fourier transform method to solve the PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^3 u}{\partial x^3} \quad ,$$

with the boundary conditions

(I) $u(x, t)$ and all of its partial derivatives in x vanish as $x \to \pm\infty$
(II) $u(x,0) = \exp(-x^2)$.

Plot the solution, $u(x, t)$, as a function of t at $t = 0$, 0.1, and 0.3. Collect all three curves in one plot. It is recommended that the plot be made to cover the interval of $-10 \leq x \leq 10$. For this problem, it is acceptable that your solution be expressed as an integral. The values of $u(x, t)$ that are needed to make the plot can be obtained by numerical integration.

Prob 2 (2 points)
For $u(x,t)$ defined on the infinite domain, $-\infty < x < \infty$, and $t \geq 0$, use the Fourier transform method to solve the PDE

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} - 2u \quad ,$$

with the boundary conditions

(I) $u(x, t)$ and all of its partial derivatives in x vanish as $x \to \pm\infty$
(II) $u(x,0) = \exp(-x^2)$.

For this problem, the goal is to obtain a closed-form analytic solution. All integrals that appear in the intermediate steps should be evaluated analytically.

Hint: For both problems, you may find the following formula useful:

$$\int_0^\infty e^{-x^2} \cos(2bx)\, dx = \frac{\sqrt{\pi}}{2} e^{-b^2} \ .$$

More precisely, it is useful for carrying out the Fourier transform in Prob 1, and both the F.T. and inverse F. T. in Prob 2.