MAE/MSE502 Spring 2014 Homework #6B

Prob 1 (3 points)
For $u(x, t)$ defined on the infinite domain, $-\infty < x < \infty$, and $t \geq 0$, consider the PDE,

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + u\,\frac{\partial u}{\partial x} = 0.5 \quad ,$$

with the boundary condition,

$$u(x, 0) = \mathrm{P}(x) \ ,$$

where

$$\begin{aligned} \mathrm{P}(x) &= 1 \quad &&, \text{ if } x < 0 \\ &= 1 - x &&, \text{ if } 0 \leq x \leq 1 \\ &= 0 &&, \text{ if } x > 1 \ . \end{aligned}$$

(a) Find the analytic solution that is valid for $t < 1$. Plot the solution as a function of x at $t = 0$ (initial state), 0.5, and 0.8.
(b) Evaluate $u(x, t)$ at ($x = 0.9$, $t = 0.4$) and ($x = 1.6$, $t = 0.5$).
(c) Plot selected characteristics of this problem in the x-t plane and use them to discuss the behavior of the solution. In particular, discuss the behavior of the solution as $t \to 1$.

Prob 2 (2 points)
For $u(x, t)$ defined on the infinite interval, $-\infty < x < \infty$, and for $t \geq 0$, solve the PDE,

$$\frac{\partial u}{\partial t} - t\frac{\partial u}{\partial x} - x = 0 \quad ,$$

with the boundary condition,

$$u(x, 0) = \mathrm{P}(x) \ ,$$

where

$$\begin{aligned} \mathrm{P}(x) &= 0 \quad &&, \text{ if } x < 0 \\ &= x^2 &&, \text{ if } 0 \leq x \leq 1 \\ &= 1 &&, \text{ if } x > 1 \end{aligned}$$

Plot the solution as a function of x at $t = 0$ (initial state), 0.5, and 1.

Prob 3 (2 points)
For $u(x, t)$ defined on the infinite interval, $-\infty < x < \infty$, and for $t \geq 0$, find the solution of the PDE,

$$0.5\,\frac{\partial u}{\partial t} + x\,(\frac{\partial u}{\partial x} + 1) = 0 \quad ,$$

with the boundary condition,

$$u(x, 0) = \mathrm{P}(x) \ ,$$

where

$$\begin{aligned} \mathrm{P}(x) &= 1 \quad &&, \text{ if } x \leq 0 \\ &= e^{-x} &&, \text{ if } x > 0 \end{aligned}$$

Plot the solution as a function of x at $t = 0$ (initial state), 0.1, and 0.3.