

MAE/MSE 502 (Fall 2015) HW1 Solutions

Prob 1

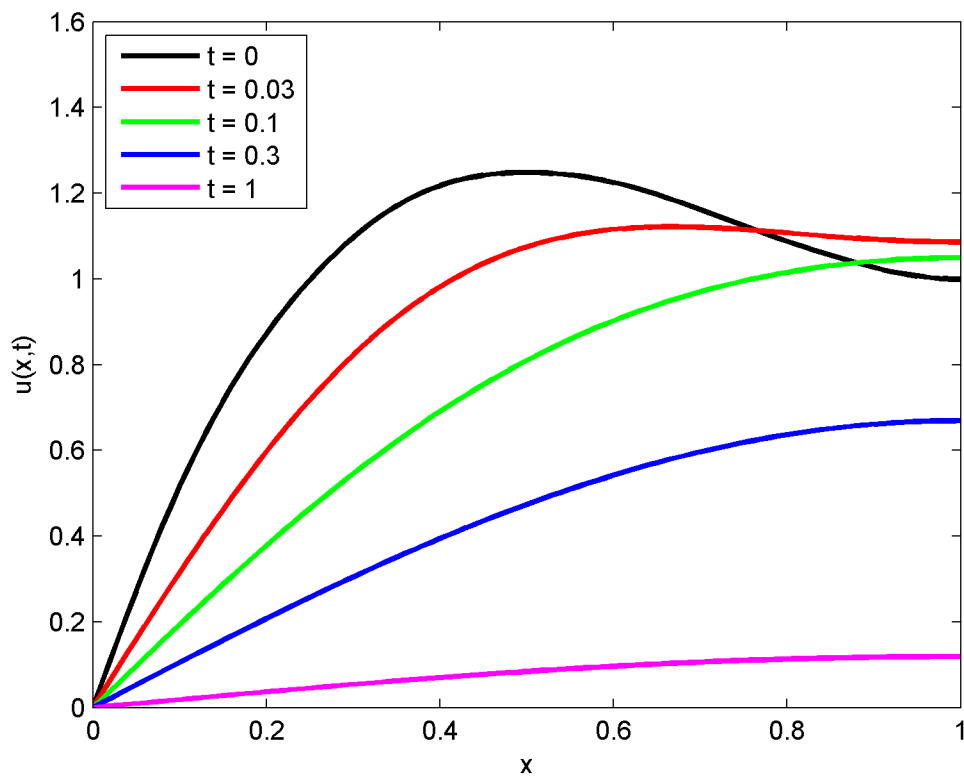
$$u(x, t) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{2}\right) \exp\left(-\left(\frac{n\pi}{2}\right)^2 t\right),$$

with the summation carried out over **odd values of  $n$  only**, and with

$$a_n = \frac{\int_0^1 \sin\left(\frac{n\pi x}{2}\right) (4x^3 - 9x^2 + 6x) dx}{\int_0^1 \left[\sin\left(\frac{n\pi x}{2}\right)\right]^2 dx}, \text{ for odd values of } n.$$

The expansion coefficient,  $a_n$ , can be evaluated analytically or numerically using Matlab.

Plot of solution (with the infinite series truncated at  $n = 15$ ):



Prob 2

$$u(x, t) = \sin\left(\frac{\pi x}{2}\right) \exp\left(-\left(\frac{\pi}{2}\right)^2 [2t + 1 - \cos(t)]\right) + 2 \sin\left(\frac{3\pi x}{2}\right) \exp\left(-\left(\frac{3\pi}{2}\right)^2 [2t + 1 - \cos(t)]\right)$$

Prob 3

The steady state solution is  $u_s(x) = \frac{7}{120}$  .

Prob 4

We will discuss the solution in class.