MAE/MSE502 Fall 2015 Homework #3

Prob. 1 (1.5 points)
For $u(x, t)$ defined on the domain of $0 \le x \le 4$ and $t \ge 0$, consider the 1-D Wave equation,

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \quad ,$$

with the boundary conditions,

(i) $u(0, t) = 0$, (ii) $u(4, t) = 0$, (iii) $u(x, 0) = \mathrm{P}(x)$, (iv) $u_t(x, 0) = 0$.

Solve the system for the two cases with

(a) $\mathrm{P}(x) = \sqrt{x} - \dfrac{x}{2}$

(b) $\mathrm{P}(x) = x$, if $0 \le \mathrm{x} \le 1$
$= (4 - x)/3$, if $1 < \mathrm{x} \le 4$

For each case, plot the solution as a function of x at t = 0, 1.2, 2, 2.8, 4, and 7.2. Please collect all 6 curves in one plot.

Prob. 2 (3 points)
For $u(x, t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, solve the PDE,

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + 1 + \mathrm{e}^{-t}\sin(2x) \quad ,$$

with the boundary conditions,

(i) $u(0, t) = u(2\pi, t)$
(ii) $u_x(0, t) = u_x(2\pi, t)$
(iii) $u(x, 0) = 2$
(iv) $u_t(x, 0) = 3 + \cos(x)$.

We expect a closed form solution with no unevaluated integral or summation of infinite series. Otherwise, no need to make any plot.

Prob 3 (1.5 points)
For $u(x, y, t)$ defined on the domain of $x^2 + y^2 \leq 1$ (a circular disk, see figure below) and $t \geq 0$, consider the modified 2-D Heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - u \quad ,$$

with the boundary conditions,

(i) $\hat{n} \cdot \nabla u = 2$, at $x^2 + y^2 = 1$ (i.e., the circular boundary) where $\hat{n}$ is the outward unit normal vector at the boundary.

(ii) $u(x, y, 0) = x^2 + y^2$.

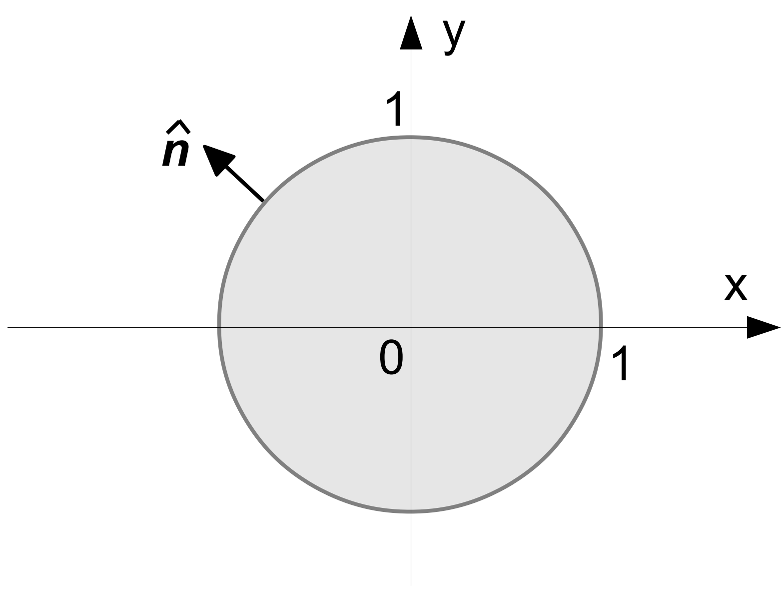
The total energy of the system is defined as

$$E(t) \equiv \iint_A u(x, y, t) \, dA \quad ,$$

where the integral is over the whole circular domain.

Use the given information to evaluate $E(t)$ at $t = 1$ and as $t \to \infty$.

[Note: For this problem, all you are asked to do is to evaluate $E(t)$. You may or may not need to find the full solution, $u(x, y, t)$.]



Prob 4 (3 points)

For $u(x, y, t)$ defined on the domain of $0 \le x \le 1$, $0 \le y \le 1$, and $t \ge 0$, solve the PDE

$$(1+t)\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} - 4\frac{\partial^2 u}{\partial y^2} = 0 \quad ,$$

with the boundary conditions,

(i) $u_x(0, y, t) = 0$
(i) $u_x(1, y, t) = 0$
(i) $u(x, 0, t) = 0$
(i) $u(x, 1, t) = 0$
(i) $u(x, y, 0) = \sin(\pi y)[2 + \cos(\pi x)]$.

We expect a closed form solution with no unevaluated integral or summation of infinite series.