

Prob 1

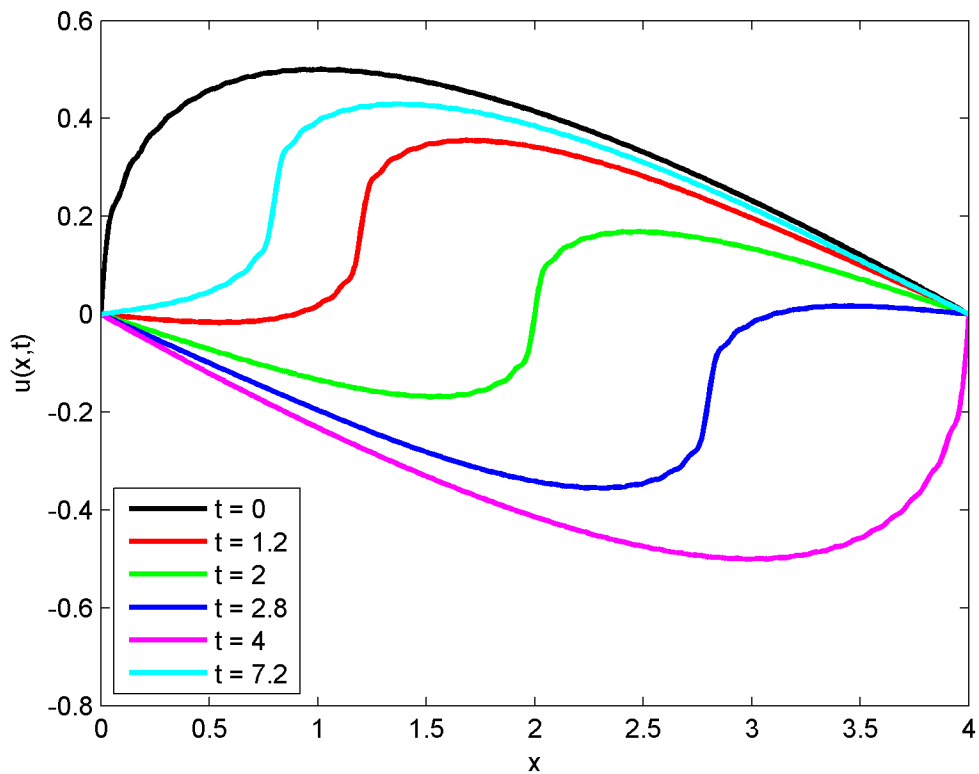
The solution is

$$u(x, t) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{4}\right) \cos\left(\frac{n\pi t}{4}\right),$$

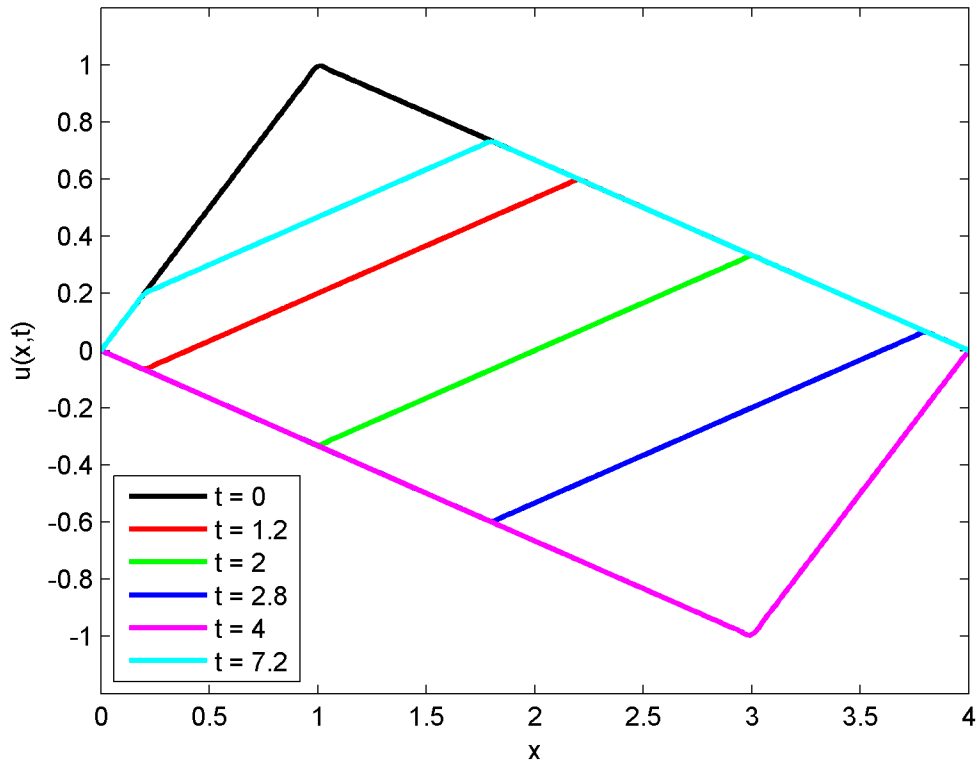
where

$$a_n = \frac{\int_0^4 P(x) \sin\left(\frac{n\pi x}{4}\right) dx}{\int_0^4 \left[\sin\left(\frac{n\pi x}{4}\right)\right]^2 dx}.$$

Plot for Part (a) (with 90-term truncation)



Plot for Part (b) (with 90-term truncation)



Prob 2

$$u(x, t) = \left(2 + 3t + \frac{t^2}{2}\right) + \sin(t) \cos(x) + \frac{1}{5} \left[\frac{1}{2} \sin(2t) - \cos(2t) + e^{-t}\right] \sin(2x)$$

Prob 3

$$E(t) = \frac{\pi}{2} e^{-t} + 4\pi (1 - e^{-t}) ; \quad E(1) = \frac{\pi}{2} e^{-1} + 4\pi (1 - e^{-1}) , \quad E(\infty) = 4\pi$$

Prob 4

$$u(x, y, t) = 2 \sin(\pi y) (1+t)^{-4\pi^2} + \sin(\pi y) \cos(\pi x) (1+t)^{-5\pi^2}$$