

MAE/MSE 502, Fall 2015, HW4 Solutions

Prob 1

$$u(x, t) = \cos(x-t) \exp(-0.1 t) + \sin(2x-8t) \exp(-1.6 t)$$

Prob 2

$$u(x, t) = 2 \int_0^{\infty} [U_R \cos(\omega x) - U_I \sin(\omega x)] d\omega ,$$

where U_R and U_I are the real and imaginary parts of $U(\omega, t)$ given as

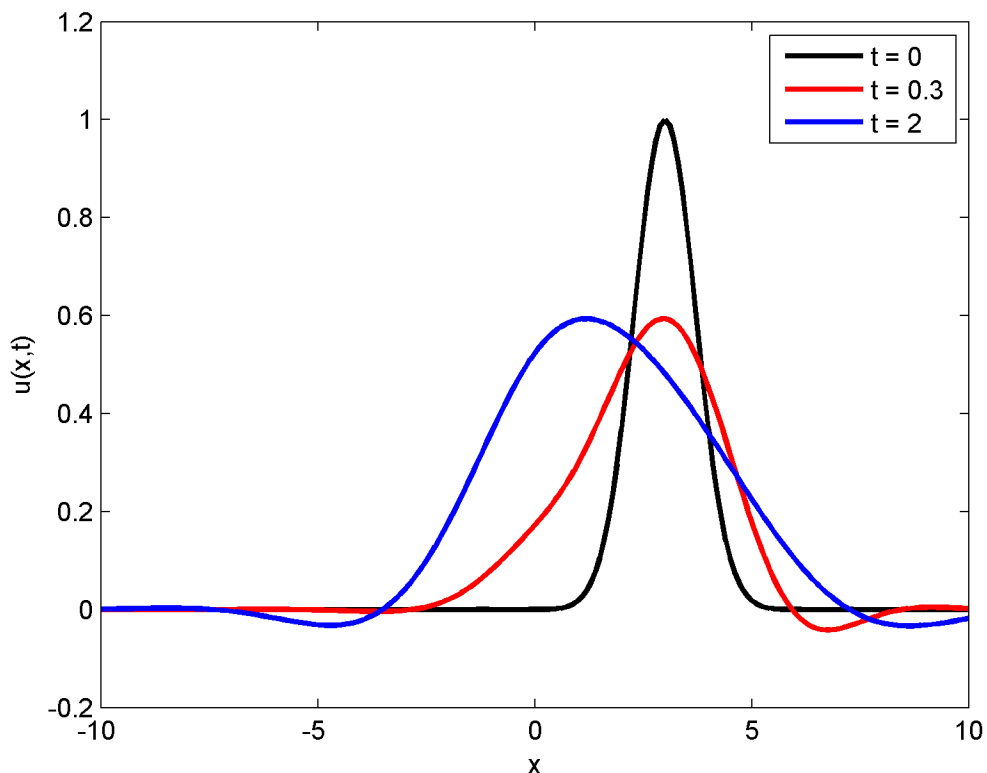
$$U(\omega, t) = U(\omega, 0) \exp(-\omega^4 t) + \frac{\gamma [\exp(-t) - \exp(-\omega^4 t)]}{\omega^4 - 1} , \text{ if } \omega \neq \pm 1$$

$$= U(\omega, 0) \exp(-\omega^4 t) + \gamma t \exp(-\omega^4 t) , \text{ if } \omega = \pm 1 ,$$

and

$$U(\omega, 0) = \frac{1}{2\sqrt{\pi}} \exp[-(\frac{\omega^2}{4} + i 3\omega + \omega^4 t)] , \quad \gamma = \frac{1}{2\sqrt{\pi}} \exp(-\frac{\omega^2}{4}) .$$

Plot



Prob 3

$$u(x, t) = \frac{1}{(2t+1)} \exp\left[-\left(\frac{x}{2t+1}\right)^2\right]$$

Prob 4

$$u(x, t) = \frac{1}{(1+t)} \exp[-(x+3t)^2]$$