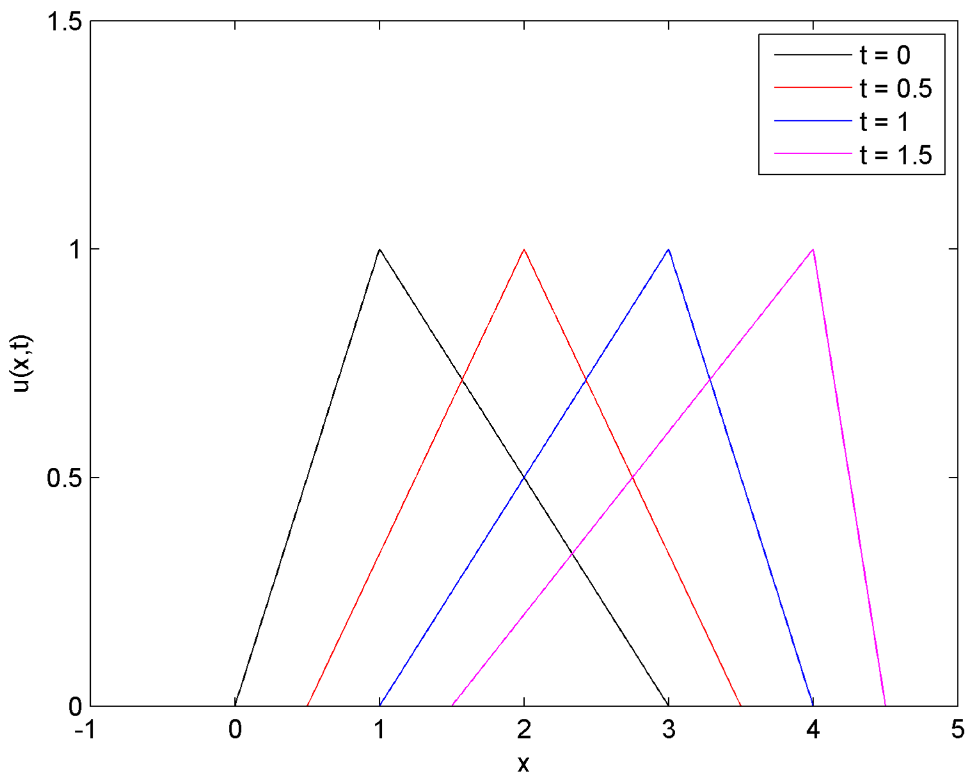
MAE/MSE502, Fall 2015, HW5 Solutions

Prob 1

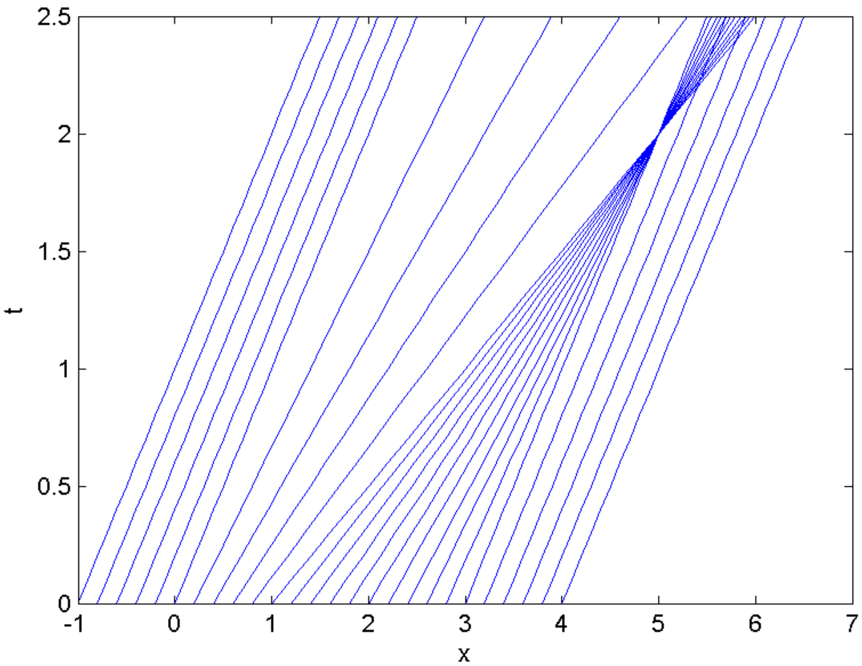
$$
\begin{aligned}
u(x,t) &= \frac{x-t}{1+t} \quad , \text{ if } t \le x < 1+2t \\
&= \frac{1}{2}\left(3 - \frac{x-2.5t}{1-0.5t}\right) \quad , \text{ if } 1+2t \le x \le 3+t \\
&= 0 \quad , \text{ otherwise }.
\end{aligned}
$$

Finite-time blowup occurs when the second segment with $1+2t \le x \le 3+t$ becomes vertical, i.e., its "shadow" on the x-axis is reduced to a point. This means $1+2t = 3+t$. So, the critical time is $\underline{t=2}$.

Plot of solution:



Prob 1, Plot of characteristics:

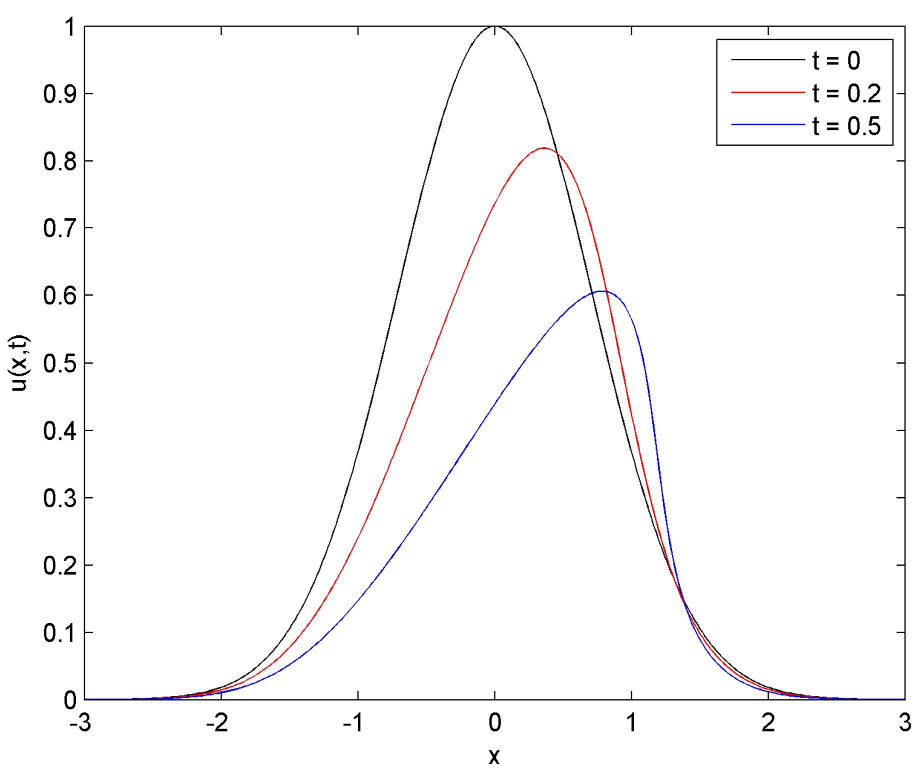


Prob 2

$u(x,t) = \exp(-x_0^2 - t)$, where x_0 is determined numerically from

$x = x_0 + 2\exp(-x_0^2)\,[1-\exp(-t)]$.

Plot:



Prob 3

$$u(x, y, t) = \exp[\, - \{(x \exp(-t))^2 + (y - t^2/2)^2\} + 2t\,]$$

Prob 4

$$u(x, t) = \frac{1}{1+t} \exp[-(\frac{x}{1+t})^2 + t]$$

Prob 5

$$G(t, t') = \exp[-(3t + t^2/2) + (3t' + t'^2/2)]$$