

## Solutions

### Prob 1

$$u(x, y) = u_1(x, y) + u_2(x, y) \quad ,$$

where

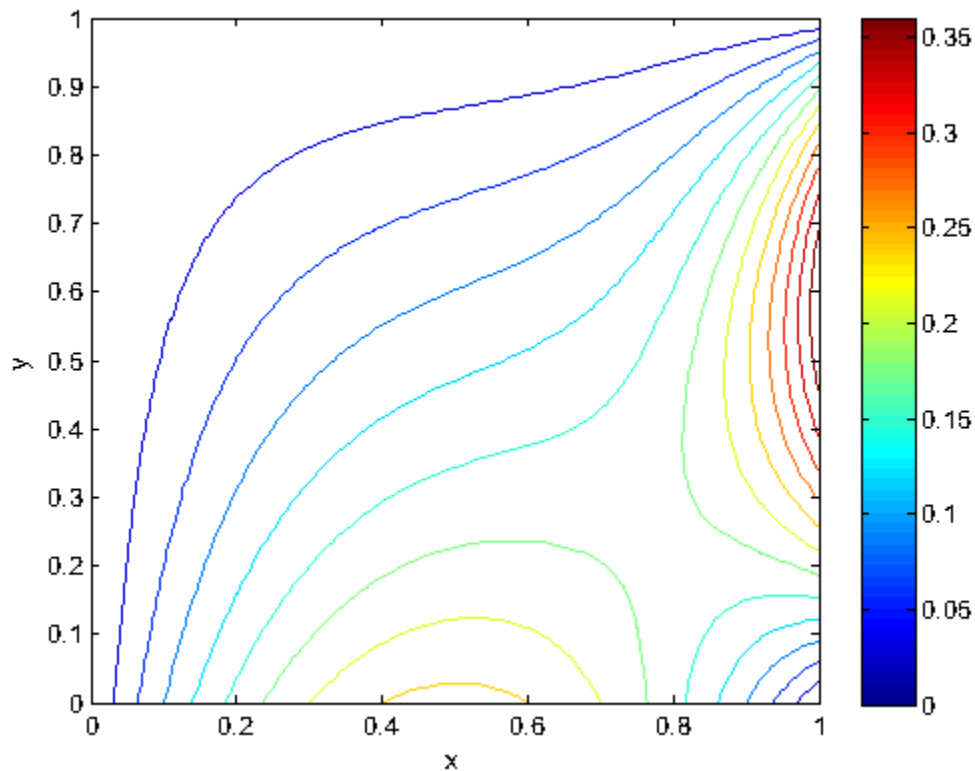
$$u_1(x, y) = \sum_{n=1}^{\infty} a_n \sin(n\pi x) \sinh\left(\frac{n\pi}{2}(1-y)\right) \quad , \quad a_n = \frac{2}{\sinh(n\pi/2)} \int_0^1 (x-x^2) \sin(n\pi x) dx \quad ,$$

and

$$u_2(x, y) = \sum_{n=1}^{\infty} b_n \sinh(2n\pi x) \sin(n\pi y) \quad , \quad b_n = \frac{2}{\sinh(2n\pi)} \int_0^1 (y-y^3) \sin(n\pi y) dy$$

$$u(0.7, 0.4) = 0.1543$$

Plot (contour interval is 0.03; color scale shown at right):



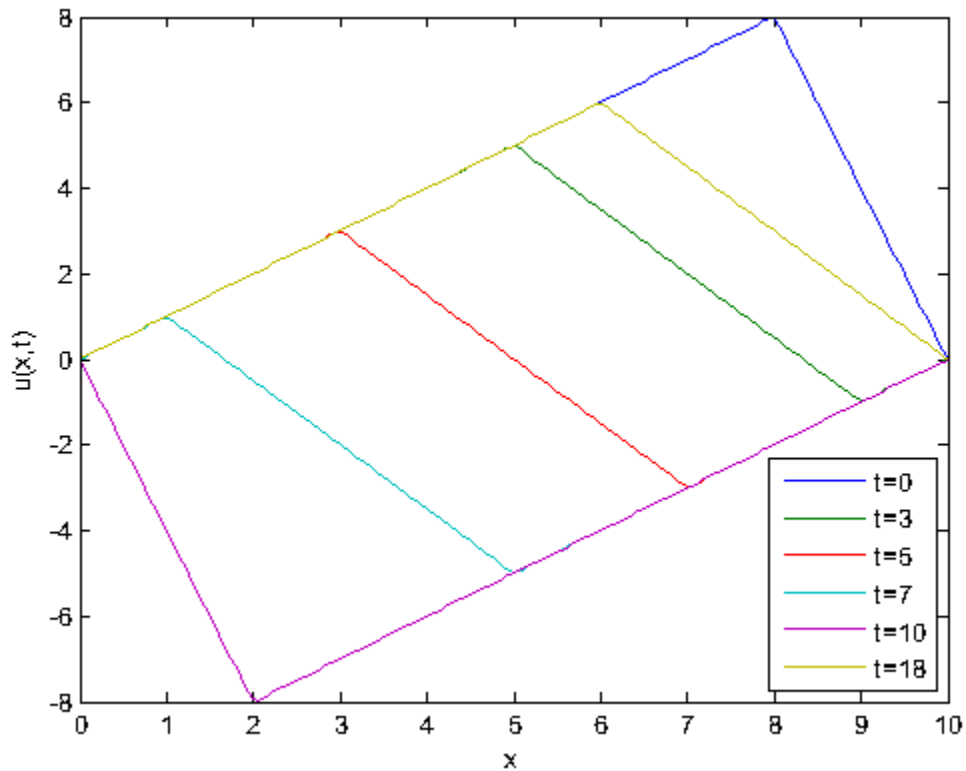
Prob 2

$$u(x, t) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{10}\right) \cos\left(\frac{n\pi t}{10}\right) ,$$

where

$$a_n = \frac{\int_0^{10} P(x) \sin\left(\frac{n\pi x}{10}\right) dx}{\int_0^{10} \sin^2\left(\frac{n\pi x}{10}\right) dx} = 0.2 \left[ \int_0^8 x \sin\left(\frac{n\pi x}{10}\right) dx + \int_8^{10} (40 - 4x) \sin\left(\frac{n\pi x}{10}\right) dx \right] .$$

Notice the factor of 0.2 (instead of 2) in front of the last integrals.



Prob 3

$$u(x, t) = \cosh(\pi t) + \frac{4}{\pi} \sinh(\pi t) + (5t + 2) \cos(\pi x) + \left[ 3 \cos(\sqrt{24}\pi t) + \frac{6}{\sqrt{24}\pi} \sin(\sqrt{24}\pi t) \right] \cos(5\pi x)$$