MAE/MSE502 Spring 2015 Homework #2

Prob 1 (4 points)
For $u(x, y)$ defined on the square domain of $0 \le x \le 1$ and $0 \le y \le 1$, solve the slightly modified Laplace's equation

$$\frac{\partial^2 u}{\partial x^2} + 4\,\frac{\partial^2 u}{\partial y^2} = 0 \quad ,$$

with the boundary conditions,

(i) $u(0, y) = 0$ (ii) $u(1, y) = y - y^3$ (iii) $u(x, 0) = x - x^2$ (iv) $u(x, 1) = 0$.

Make a contour plot of your solution. Based on the solution, evaluate $u(x, y)$ at $x = 0.7$, $y = 0.4$. (Major deduction if this value is not calculated.) See Additional Note at the end of this assignment for an example on how to make a contour plot using Matlab.

Prob 2 (2 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 10$ and $t \ge 0$, solve the Wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \quad ,$$

with the boundary conditions,

(i) $u(0, t) = 0$ (ii) $u(10, t) = 0$ (iii) $u(x, 0) = \mathrm{P}(x)$ (iv) $u_t(x, 0) = 0$ (u_t is $\partial u/\partial t$) ,

where

$$\mathrm{P}(x) = x \quad , \text{ if } 0 \le x \le 8$$
$$= 40 - 4x \, , \text{ if } 8 < x \le 10 \, .$$

Plot the solution as a function of x at $t = 0$, 3, 5, 7, 10, and 18. Please put all 6 curves in one plot.

Prob 3 (3 points)
For $u(x, t)$ defined on the domain of $0 \le x \le 1$ and $t \ge 0$, solve the PDE,

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \pi^2 \, u \quad ,$$

with the boundary conditions,

(i) $u_x(0, t) = 0$ (ii) $u_x(1, t) = 0$
(iii) $u(x, 0) = 1 + 2\cos(\pi x) + 3\cos(5\pi x)$
(iv) $u_t(x, 0) = 4 + 5\cos(\pi x) + 6\cos(5\pi x)$

For this problem, we expect a closed-form analytic solution without any unevaluated integral or infinite series. A deduction will be assessed for any such items that are left untreated in the final answer. No need to make any plot for this problem.

Additional Note: Using Matlab to make a contour plot

The following Matlab code makes a contour plot for $u(x,y) = \sin(2\pi x)\exp(-2y)$ for the domain of $0 \le x \le 1$ and $0 \le y \le 1$, using the contour levels of (–0.9, –0.7, –0.5, –0.3, –0.1, –0.05, 0.05, 0.1, 0.3, 0.5, 0.7, 0.9). The contours for $u = -0.7, -0.3, 0.3$, and 0.7 are labeled. It is essential to provide the coordinates of the grid (x2d and y2d in this example) as the input for the contour function. Without this piece of information, Matlab would not know the grid spacing and the correct directions of x and y. A black-and-white contour plot is acceptable as long as the contours are properly labeled. For Prob 1, the recommended contour interval is 0.03.

```
clear
x = [0:0.01:1]; y = [0:0.01:1];
for i = 1:length(x)
    for j = 1:length(y)
        u(i,j) = sin(2*pi*x(i))*exp(-2*y(j));
        x2d(i,j) = x(i);
        y2d(i,j) = y(j);
    end
end
[C,h] = contour(x2d,y2d,u,[-0.9:0.2:-0.1 -0.05 0.05 0.1:0.2:0.9]);
clabel(C,h,[-0.7 -0.3 0.3 0.7])
xlabel('x'); ylabel('y')
```