Solutions

Prob 1
(a)

All c ≥ 0 are eigenvalues (so we have continuous eigenvalues). The eigenfunction associated with a positive eigenvalue, c, is

$$G_c(x) = [3 - \frac{5\sinh(\sqrt{c})}{\sqrt{c}}]\frac{\cosh(\sqrt{c}\,x)}{\cosh(\sqrt{c})} + \frac{5\sinh(\sqrt{c}\,x)}{\sqrt{c}} \quad .$$

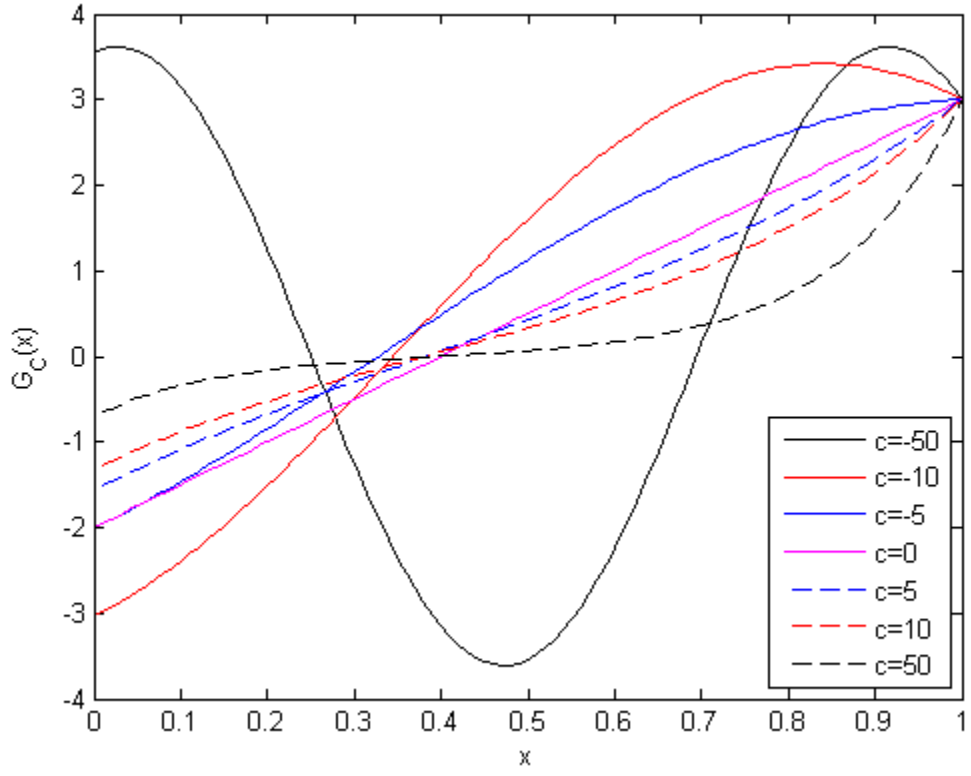
The eigenfunction associated with c = 0 is

$$G_0 = 5x - 2 \quad .$$

All c < 0, except c = - $(n\pi/2)^2$ with n = 1, 3, 5, 7, ..., are eigenvalues. The eigenfunction associated with a negative eigenvalue is

$$G_c(x) = [3 - \frac{5\sin(\sqrt{-c})}{\sqrt{-c}}]\frac{\cos(\sqrt{-c}\,x)}{\cos(\sqrt{-c})} + \frac{5\sin(\sqrt{-c}\,x)}{\sqrt{-c}} \quad .$$

(b) Plot:



(c) The eigenfunctions do not satisfy an orthogonality relation.

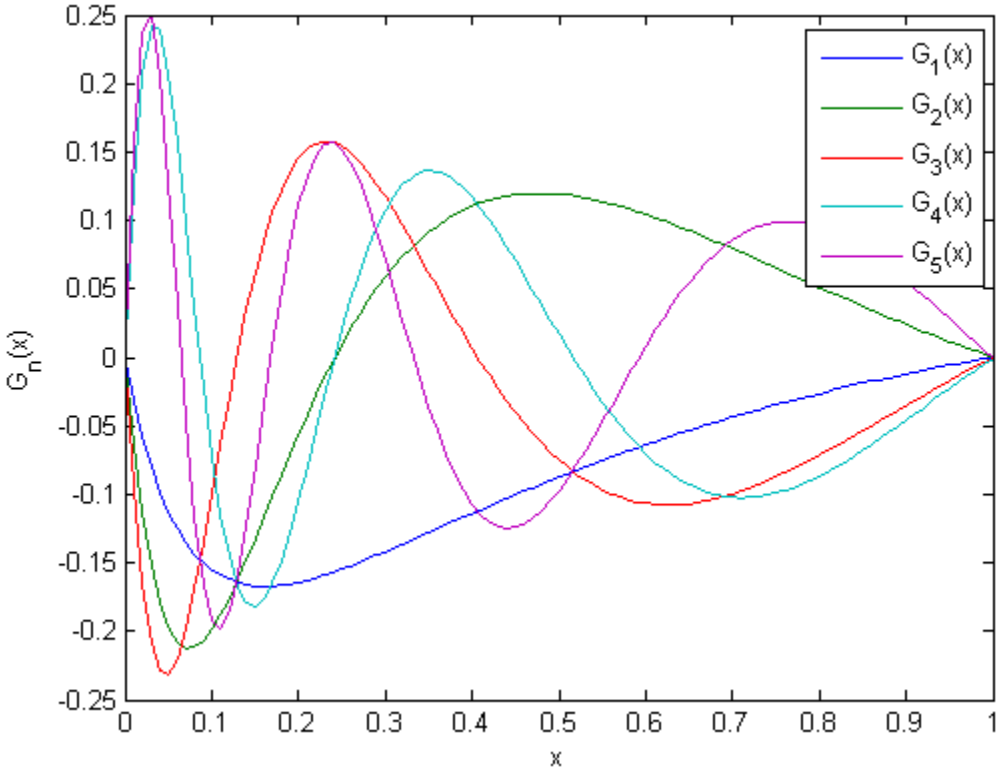
(d) No.

Prob 2

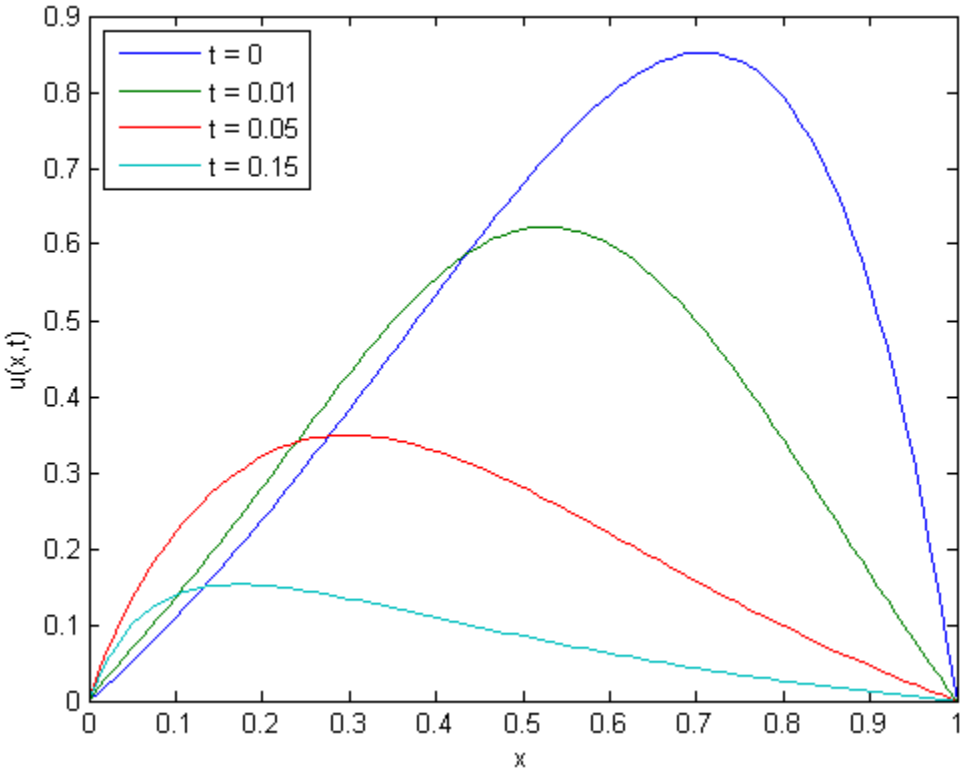
Part (a)

First five eigenvalues: -8.7581, -32.0959, -70.9251, -125.1123, -194.4768

Plot of first five eigenfunctions:



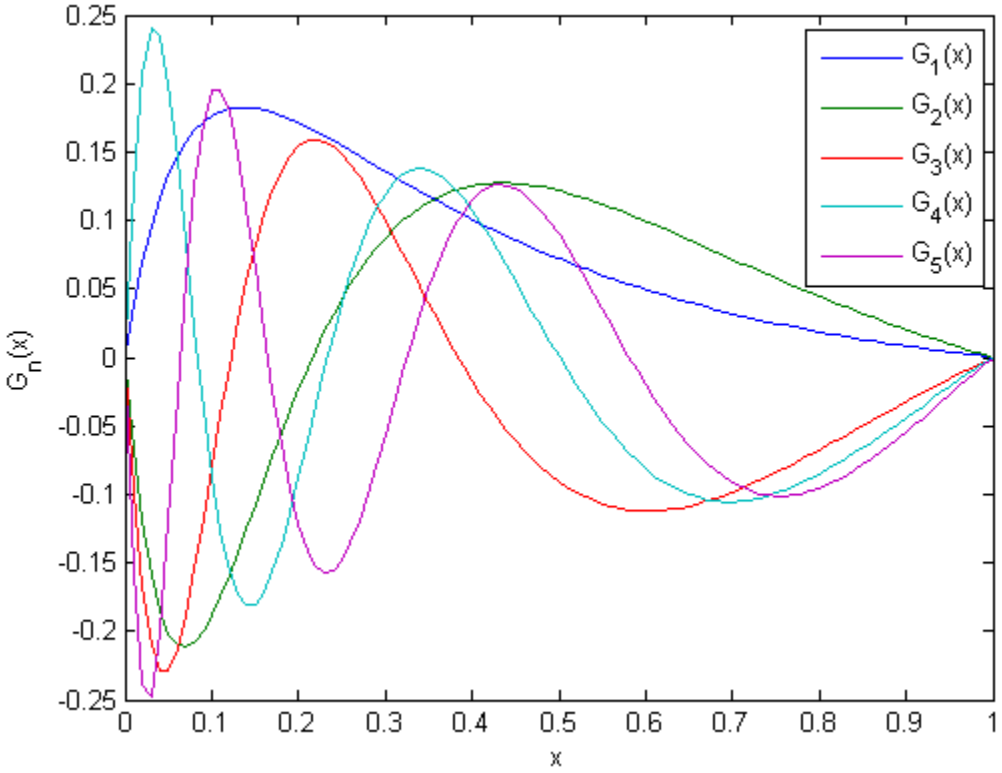
Part (a), plot of solution, u(x,t):



Part (b)

First five eigenvalues: -17.5092, -42.5389, -81.8767, -136.2361, -205.6635

Plot of first five eigenfunctions:



Part (b), plot of the solution, u(x,t):