MAE/MSE 502 Spring 2015, Homework #4

Prob. 1 (3 points) For $u(x,t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, solve the PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^3 u}{\partial x^3} \quad ,$$

with the boundary conditions (the first three simply indicate that the system is periodic in x),

(i) $u(0, t) = u(2\pi, t)$
(ii) $u_x(0, t) = u_x(2\pi, t)$
(iii) $u_{xx}(0, t) = u_{xx}(2\pi, t)$
(iv) $u(x, 0) = \sin(x) + \cos(2x)$.

We expect a closed-form solution without any unevaluated integral or summation of infinite series. Plot the solution as a function of x at $t = 0$, 0.1, 0.2, and 0.5.

Prob. 2 (3 points) For $u(x, t)$ defined on the domain of $0 \le x \le 1$ and $t \ge 0$, solve the PDE,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \cos(2\pi x)e^{-t} + t \quad ,$$

with the boundary conditions

(i) $u_x(0, t) = 0$
(ii) $u_x(1, t) = 0$
(iii) $u(x, 0) = 1 + \cos(\pi x) + \cos(2\pi x)$.

We expect a closed-form solution without any unevaluated integral or summation of infinite series.

Prob. 3 (3 points) For $u(x,t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, solve the PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^3 u}{\partial x^3} + \sin(x)e^{-t} + 1 \quad ,$$

with the boundary conditions (the first three simply indicate that the system is periodic in x),

(i) $u(0, t) = u(2\pi, t)$
(ii) $u_x(0, t) = u_x(2\pi, t)$
(iii) $u_{xx}(0, t) = u_{xx}(2\pi, t)$
(iv) $u(x, 0) = 3 + \cos(x)$.

We expect a closed-form solution without any unevaluated integral or summation of infinite series. The solution of this problem is real. Please arrange your solution such that there is no imaginary number "i" (= $\sqrt{-1}$) in the final expression of $u(x, t)$.