Solutions

Prob 1

$$u(x,t) = \frac{1}{\sqrt{\pi}} \int_0^\infty e^{-(\omega^2/4 + \omega^2 t)} \cos(\omega x + 2\omega t - 2\omega^3 t) d\omega$$

The integration can be carried out numerically.

Plot:



Prob 2

$$u(x,t) = 2\int_{0}^{\infty} Re[U(\omega,t)e^{i\omega x}]d\omega ,$$

where

$$U(\omega, t) = \frac{e^{-\omega^2/4 + \alpha(\omega)t}}{2\sqrt{\pi}} \left[1 + \frac{1 - e^{-(1 + \alpha(\omega))t}}{1 + \alpha(\omega)}\right] ,$$

and

$$\alpha(\omega) \equiv i \, 2 \, \omega - 0.2 \, \omega^2$$

The integration can be carried out numerically.

Plot:



Prob 3

 $u(x,t) = e^{-(x+5t)^2-t^2/2}$

Needs to solve the problem using the <u>Fourier transform method</u> with correct procedure to receive credit.