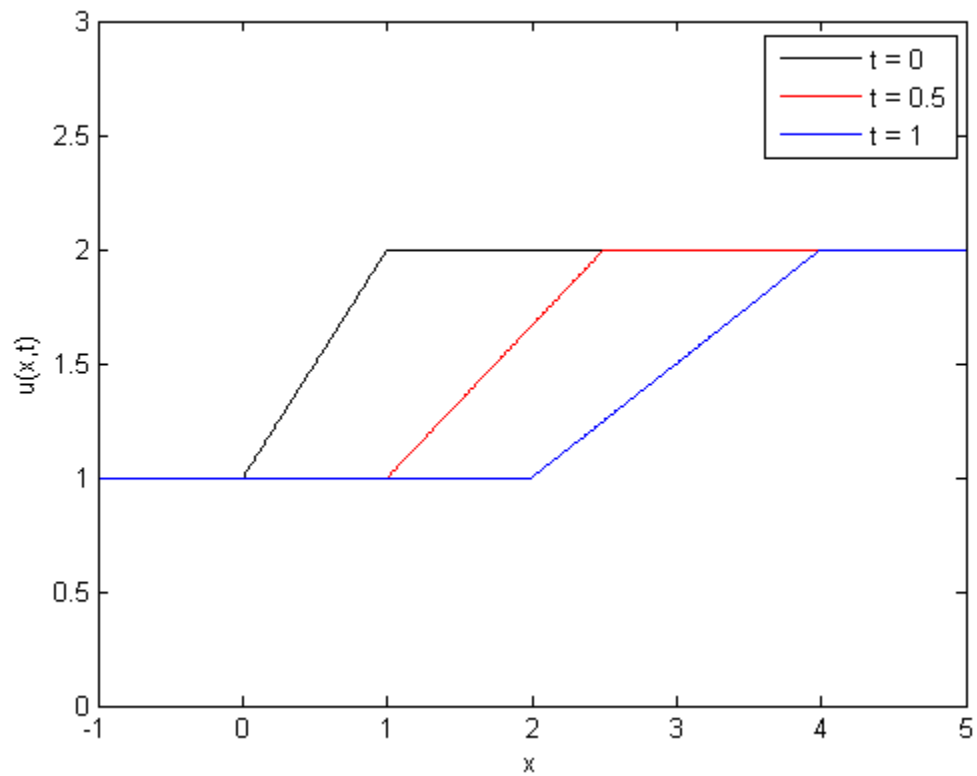


Solutions

Prob 1(a)

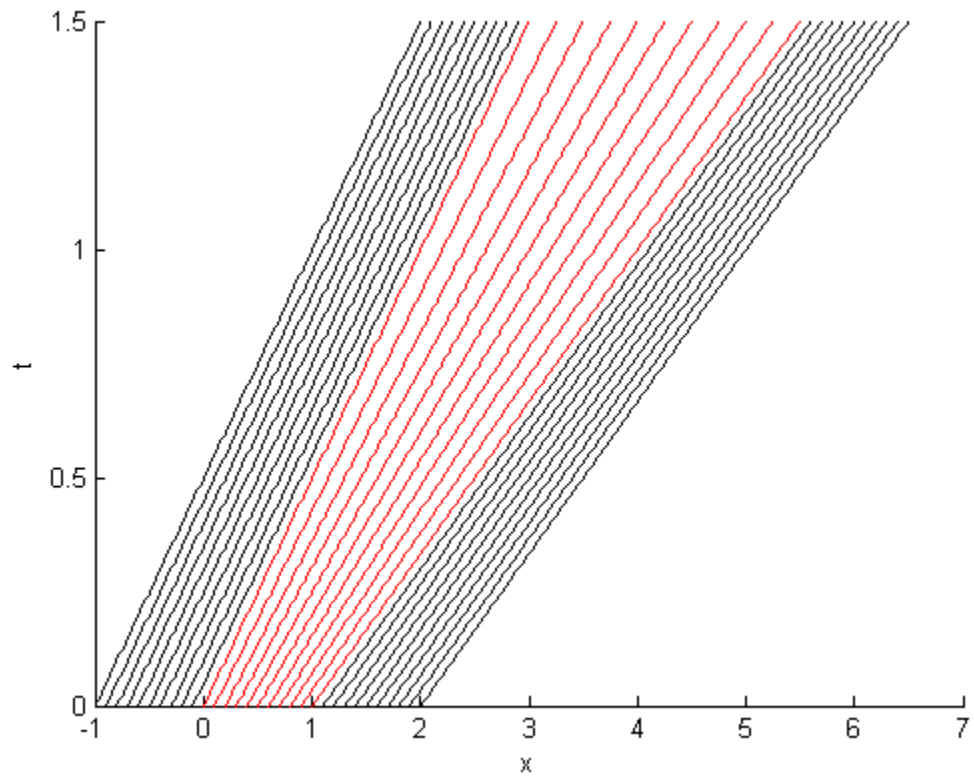
$$\begin{aligned} u(x, t) &= 1, \text{ if } x < 2t \\ &= 1 + \frac{x-2t}{1+t}, \text{ if } 2t \leq x \leq 1+3t \\ &= 2, \text{ if } x > 1+3t \end{aligned}$$

Plot of $u(x,t)$:



(Prob 1a continued)

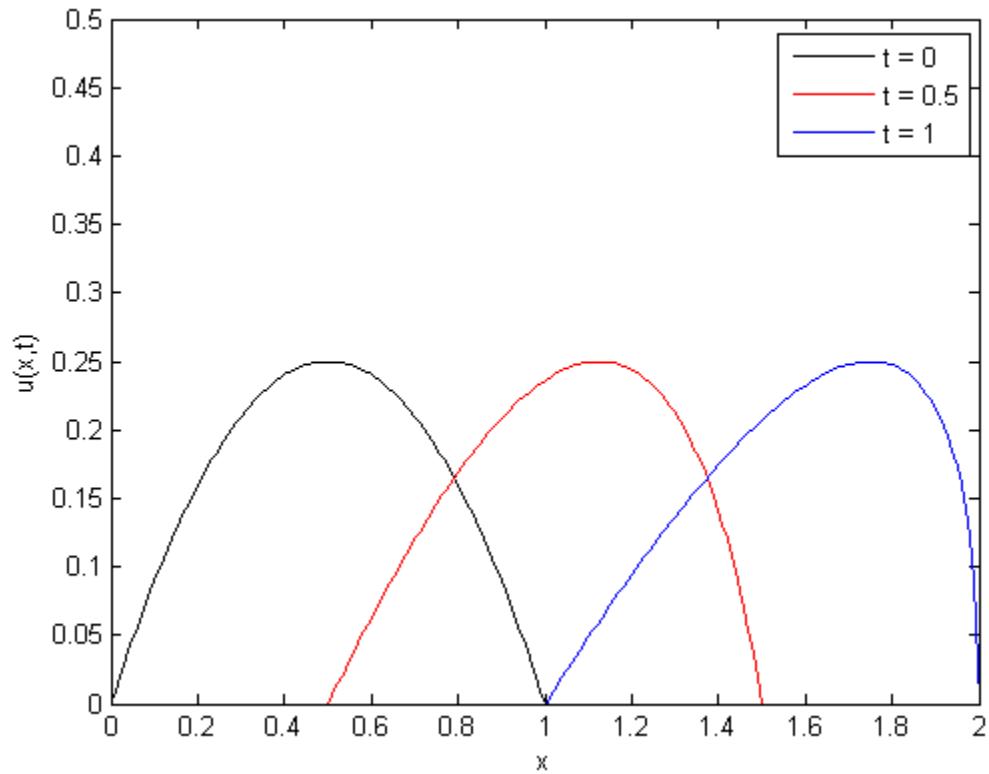
Plot of characteristics:



Prob 1(b)

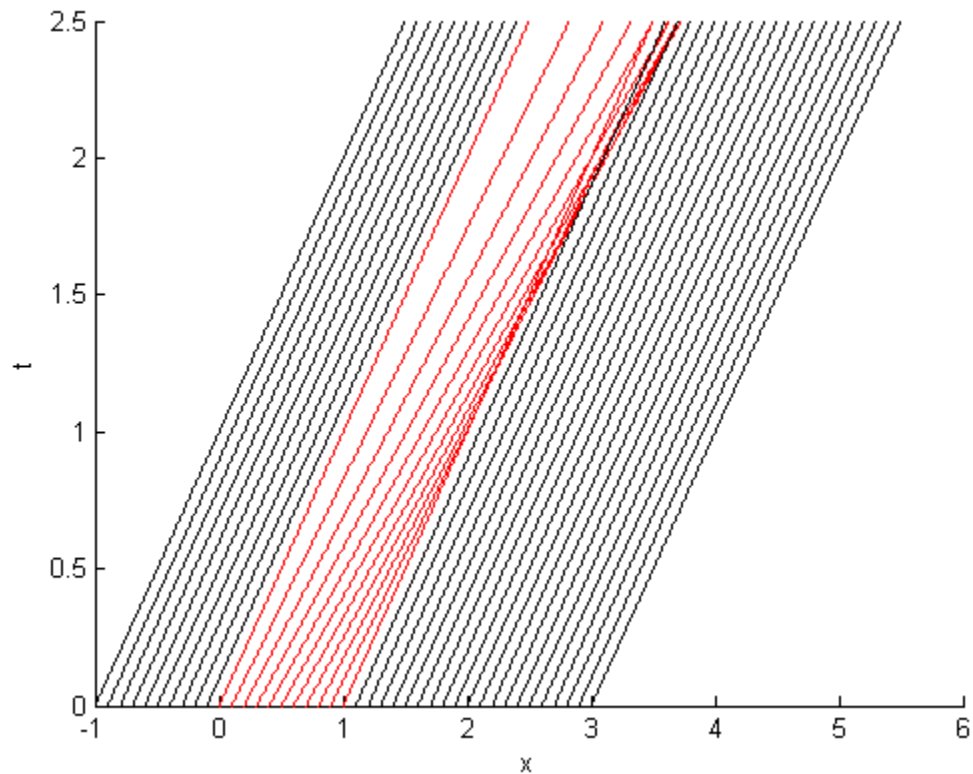
$$\begin{aligned} u(x,t) &= 0, \text{ if } x < t \\ &= x_0 - x_0^2, \text{ where } x_0 = \frac{1+t - \sqrt{(1+t)^2 - 4t(x-t)}}{2t}, \text{ if } t \leq x \leq 1+t \\ &= 0, \text{ if } x > 1+t. \end{aligned}$$

Plot of $u(x,t)$:



Prob 1b continued)

Plot of characteristics:



Prob 2

$$u(x, t) = \exp(-[x e^{-t/2}]^2) \exp([x e^{-t/2}] [e^{t/2} - 1])$$

Prob 3

$$u(x, t) = \exp(-[(x e^{-t})^2 + (y - t)^2]) + t^2/2$$

Prob 4

$$G(t, t') = \exp\left(-\frac{3}{2}(t^2 - t'^2)\right)$$