MAE/MSE 502 Spring 2016 HW1 Solutions

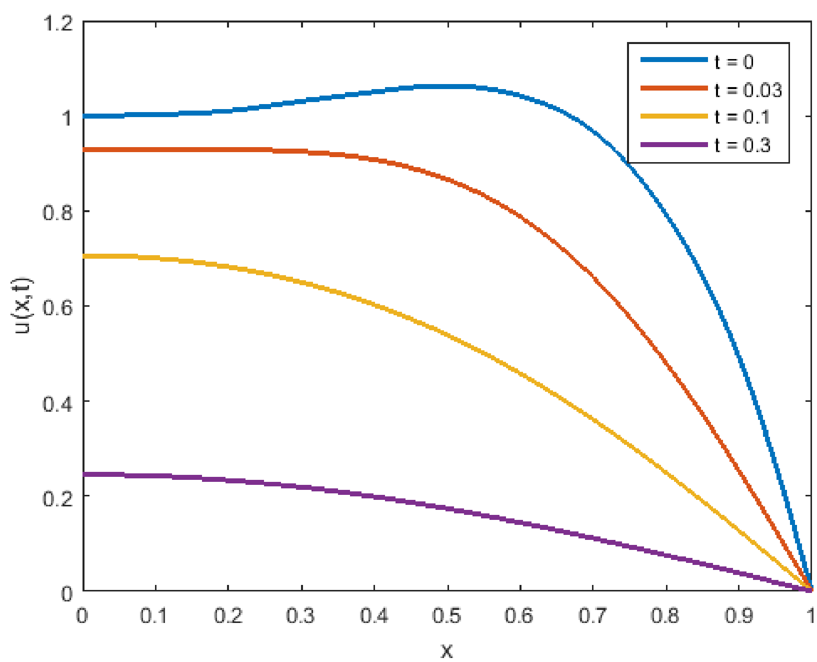
Prob 1

$$u(x,t)=\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{2}\right)\exp\left(-\left[\left(\frac{n\pi}{2}\right)^2+3\right]t\right) \quad ,$$

where the summation is carried out over odd values of $n$ only, and

$$a_n=\frac{\int_0^1 (-3x^4+2x^3+1)\cos\left(\frac{n\pi x}{2}\right)\,dx}{\int_0^1 \left[\cos\left(\frac{n\pi x}{2}\right)\right]^2\,dx}$$ , for odd values of $n$.

Plot of the solution (with the summation of infinite series truncated, inclusively, at $n$ = 19):




Prob 2

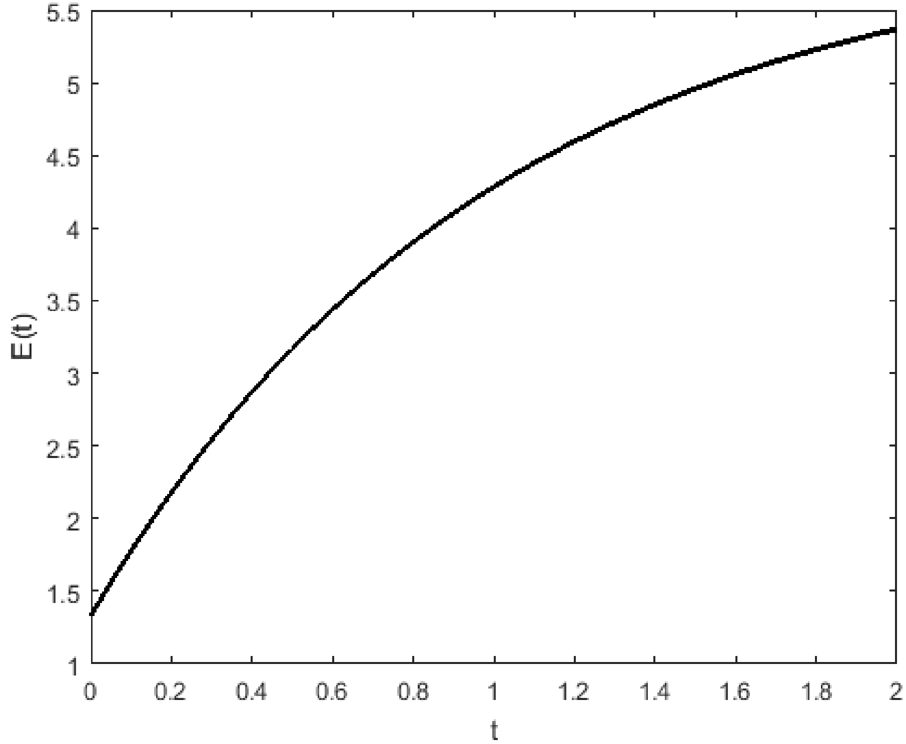
(a) $E(t) = 6 - (14/3)\exp(-t)$ (See plot in next page), $E(\infty) = 6$.

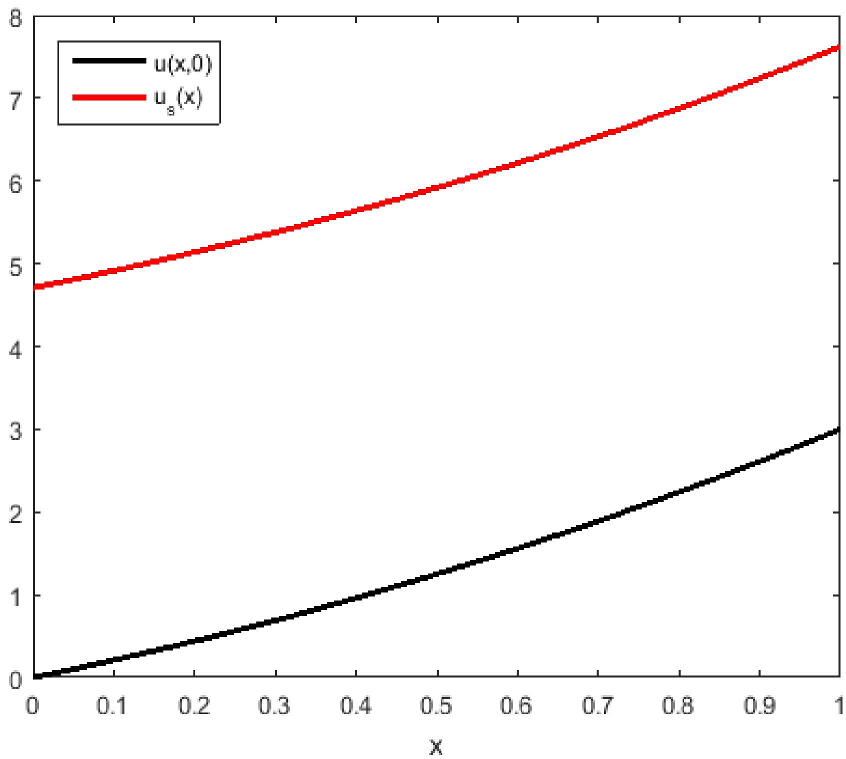
(b) Steady state solution is (see plot in next page)

$$u_s(x)=\frac{2\sqrt{3}}{\sinh\left(\frac{1}{\sqrt{3}}\right)}\left[2\cosh\left(\frac{x}{\sqrt{3}}\right)-\cosh\left(\frac{x-1}{\sqrt{3}}\right)\right]$$

(c) It can be readily shown that $\int_0^1 u_s(x)\,dx = 6$ .

Plots for Prob 2:




Prob 3

$u(x, t) = 3 \exp[1 - \cos(t)] + 4 \exp[-\pi^2 t + 1 - \cos(t)] \cos(\pi x)$

Prob 4

We will discuss the solution in class.