MAE/MSE 502, Spring 2016, HW3 Solutions

Prob 1

$$u(x,t) = 5t + 3 + t\cos(x) + \frac{1}{\sqrt{12}}\sinh(\sqrt{12}\,t)\sin(2x)$$

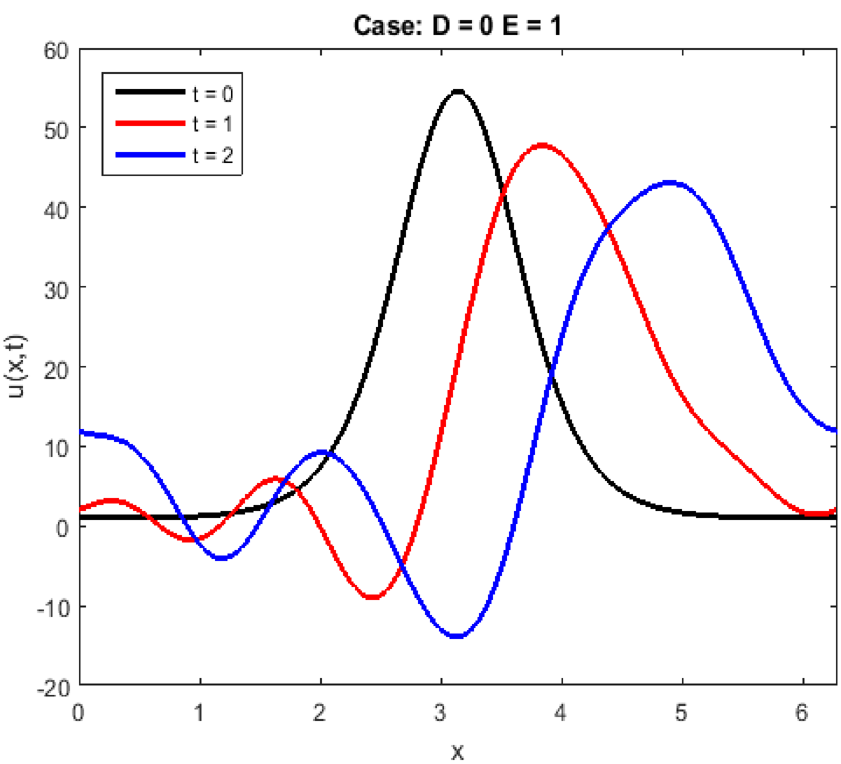
Prob 2

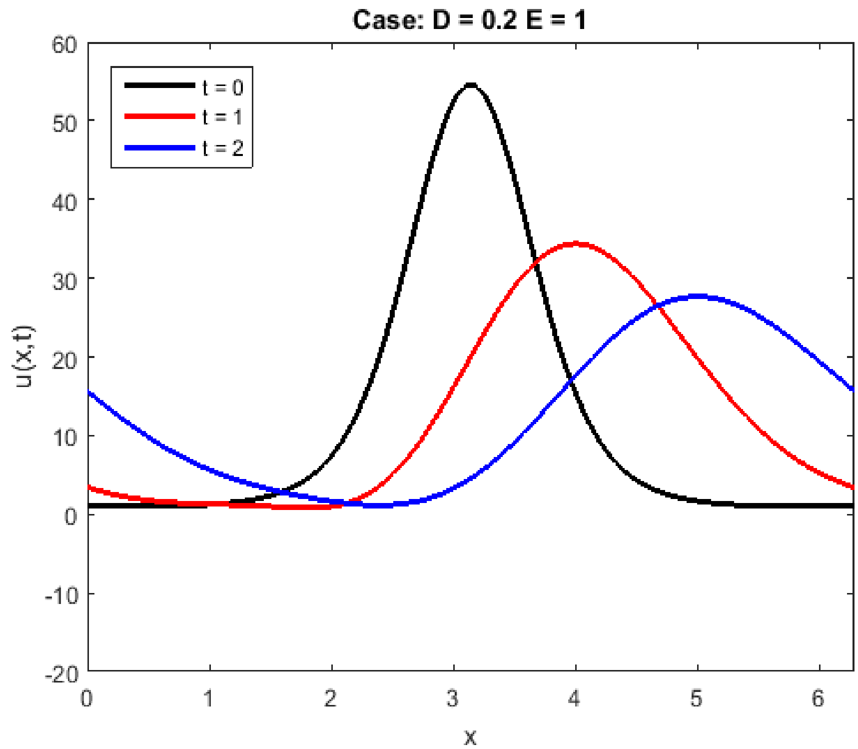
$$u(x,t) = \sum_{n=-\infty}^{\infty} C_n(0)\, e^{-n^2 D t}\, e^{in(x-n^2 E t)} \quad ,$$

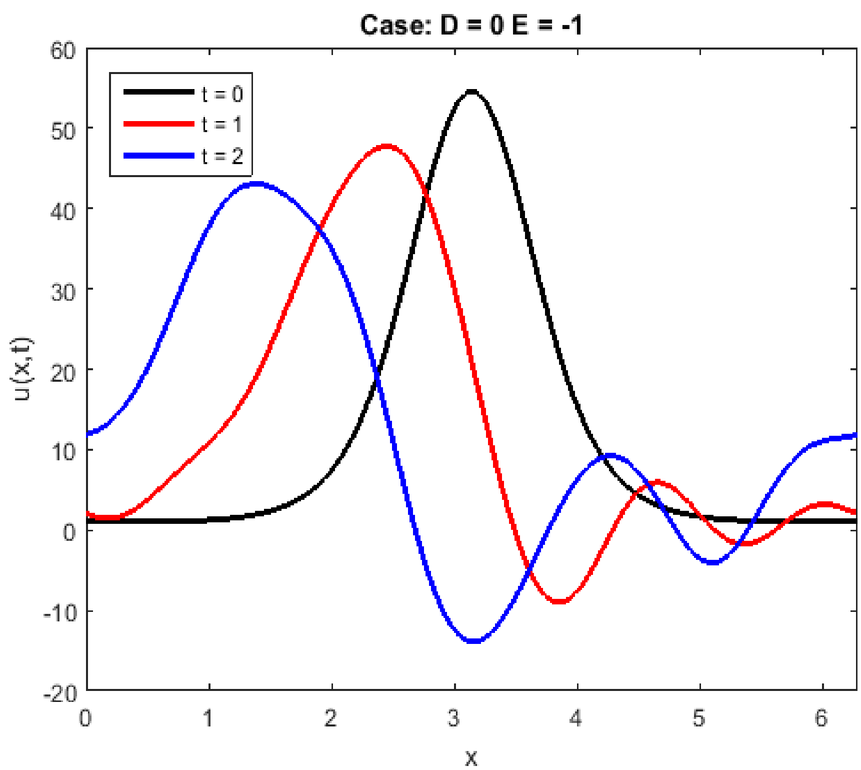
where

$$C_n(0) = \frac{1}{2\pi}\int_0^{2\pi} e^{[1-\cos(x)]^2} e^{-inx}\, dx \quad .$$

Plot:

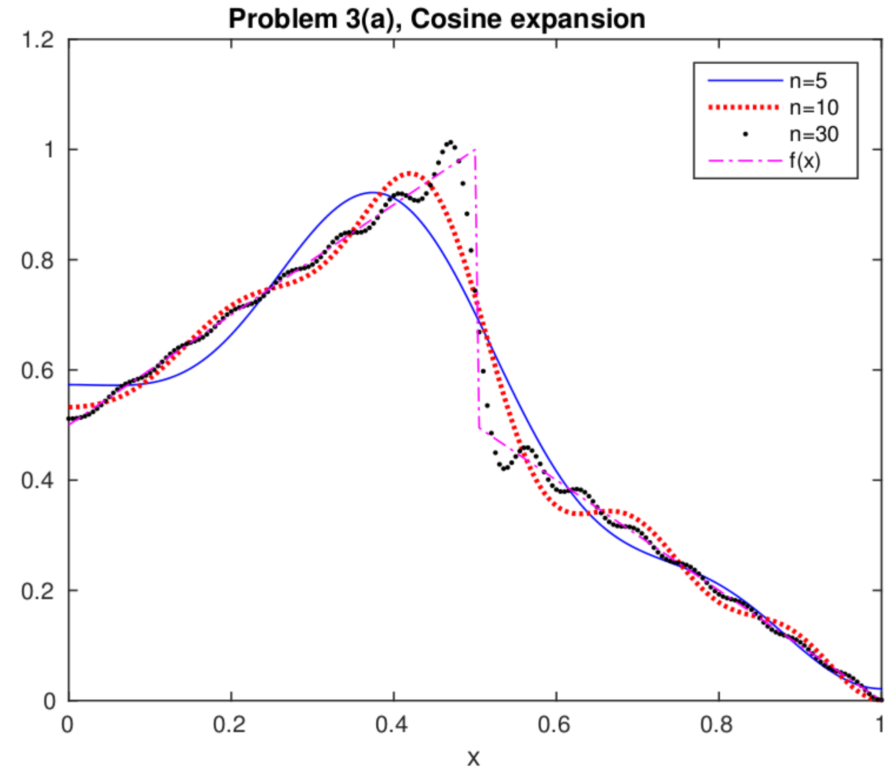



Case: D = 0.2 E = 1
t = 0
t = 1
t = 2
u(x,t)
x
60
50
40
30
20
10
0
-10
-20
0
1
2
3
4
5
6

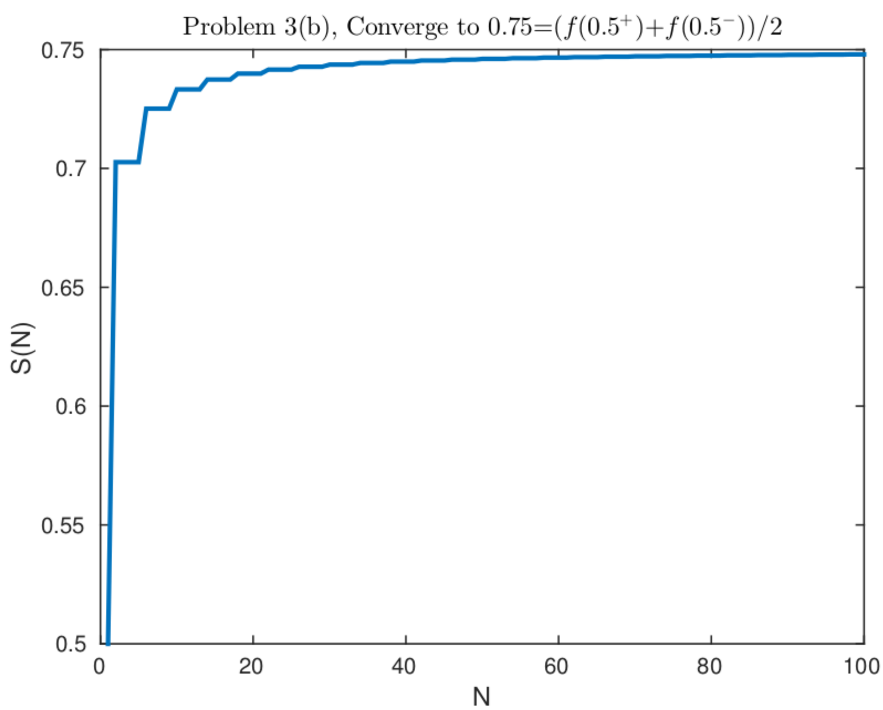

Case: D = 0 E = -1
t = 0
t = 1
t = 2
u(x,t)
x
60
50
40
30
20
10
0
-10
-20
0
1
2
3
4
5
6

Prob 3

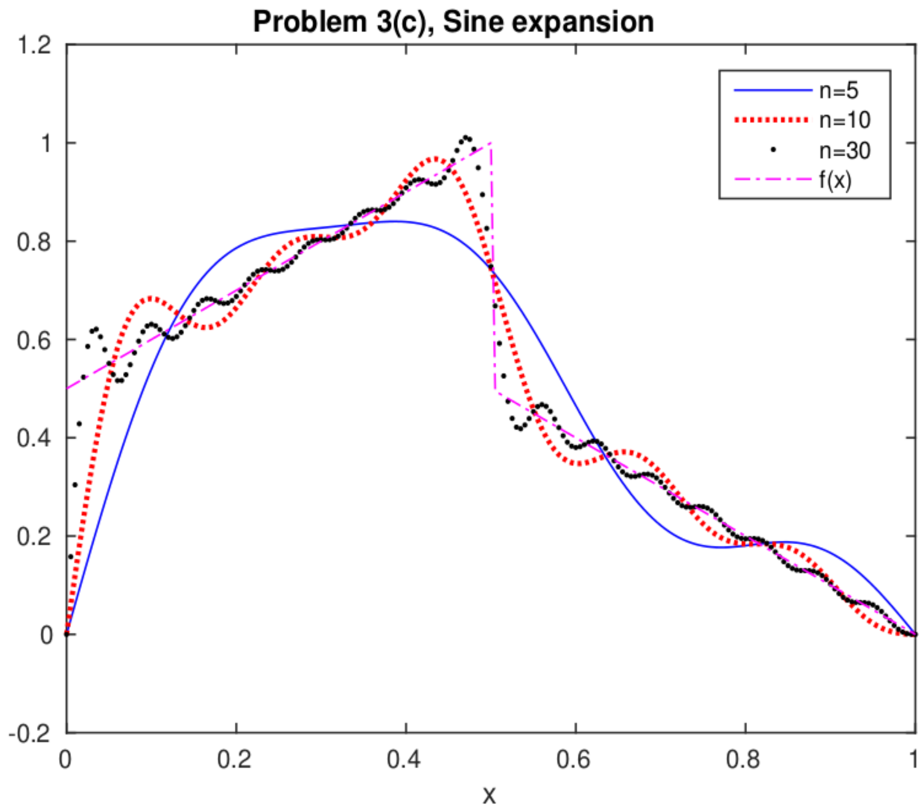
(a)


Problem 3(a), Cosine expansion
n=5
n=10
n=30
f(x)
x
0
0.2
0.4
0.6
0.8
1
1.2

(b)


Problem 3(b), Converge to 0.75=$(f(0.5^+)+f(0.5^-))/2$
S(N)
N
0.5
0.55
0.6
0.65
0.7
0.75
0
20
40
60
80
100

(c)



Fourier Cosine expansion is better, although it still has Gibbs ripples in the vicinity of $x = 0.5$ due to the discontinuity of $f(x)$ there. Fourier Sine expansion has Gibbs ripples in the vicinities of both $x = 0.5$ and $x = 0$, because the odd extension of the original $f(x)$ has a discontinuity at $x = 0$.

Prob 4

(a) $F(\omega) = (2/\omega\pi)\,[1 - \cos(\omega)]$

(b) $A_n = (2/n\pi)[1 - \cos(n\pi/L)]$, $n = 1, 2, 3, \ldots$

See plot below. This example illustrates the correspondence between ω and $n\pi/L$ in Fourier transform and Fourier series. Note that in all three cases in (b), the designated range for plotting is equivalent to $1 \leq n\pi/L < 30$. That is why they all look like the plot for $F(\omega)$.