MAE/MSE 502, Spring 2016, Homework #4

You might find the following formulas useful for this homework:

$$\int_0^\infty e^{-ax}\cos(bx)\,dx = \frac{a}{a^2+b^2}\text{ , for } a > 0.$$

$$\int_0^\infty e^{-x^2}\cos(2bx)\,dx = \frac{\sqrt{\pi}}{2}e^{-b^2}\ .$$

Prob 1 (2 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, use the method of Fourier transform to solve the PDE

$$\frac{\partial u}{\partial t} = 2\frac{\partial u}{\partial x} - 3t^2 u \quad ,$$

with the boundary conditions:

(i) $u(x, t)$ and its partial derivatives in x vanish as $x \to \pm\infty$
(ii) $u(x,0) = \exp(-x^2)$.

To receive full credit, the final solution should have a closed-form expression of a *real* function that contains no unevaluated integrals. Plot the solution $u(x,t)$ as a function of x at $t = 0$, 0.5, and 1. Please collect all three curves in one plot.

Prob 2 (3 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$(1+t)\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad ,$$

with the boundary conditions:

(i) $u(x, t)$ and its partial derivatives in x vanish as $x \to \pm\infty$
(ii) $u(x,0) = \exp(-|x|)$. ($|x|$ is the absolute value of x.)

Plot the solution $u(x, t)$ as a function of x at $t = 0$, 0.2, and 1. Please collect all three curves in one plot. It is recommended that the plot be made over the interval of $-3 \leq x \leq 3$. For this problem, it is acceptable to express the solution as an integral and use numerical integration to make the plot.

Prob. 3 (3 points)
For $u(x, t)$ defined on the domain of $0 \le x \le 1$ and $t \ge 0$, solve the PDE,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - t\,u + [1+\cos(\pi x)]\exp(-t^2/2) \quad ,$$

with the boundary conditions,

(i) $u_x(0, t) = 0$
(ii) $u_x(1, t) = 0$
(iii) $u(x, 0) = 3 + 2\cos(\pi x)$

We expect a closed form solution that contains only a finite number of terms and with no unevaluated integrals.

Prob. 4 (2 points)
For $u(x, t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, solve the PDE,

$$\frac{\partial^2 u}{\partial t^2} = 9\frac{\partial^2 u}{\partial x^2} + \frac{\partial^4 u}{\partial x^4} + 2t + \cos(x) + \sin(3x)\exp(-t) \quad ,$$

with the boundary conditions (the first four simply indicate that the system is periodic in the x-direction),

(i) $u(0, t) = u(2\pi, t)$
(ii) $u_x(0, t) = u_x(2\pi, t)$
(iii) $u_{xx}(0, t) = u_{xx}(2\pi, t)$
(iv) $u_{xxx}(0, t) = u_{xxx}(2\pi, t)$
(v) $u(x, 0) = 0$
(vi) $u_t(x, 0) = 0$

We expect a closed form solution that contains only a finite number of terms and with no unevaluated integrals.