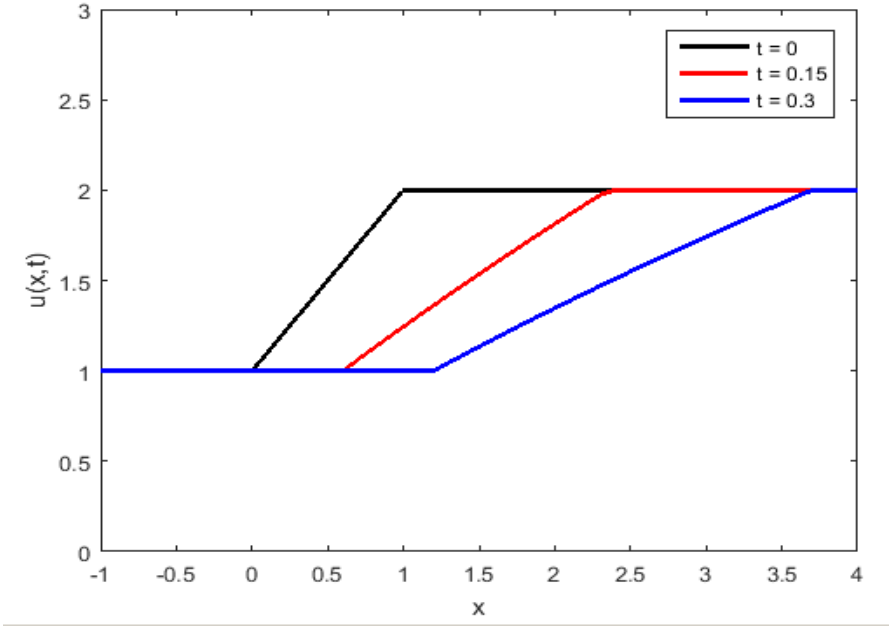
MAE/MSE 502, Spring 2016, HW5 Solutions

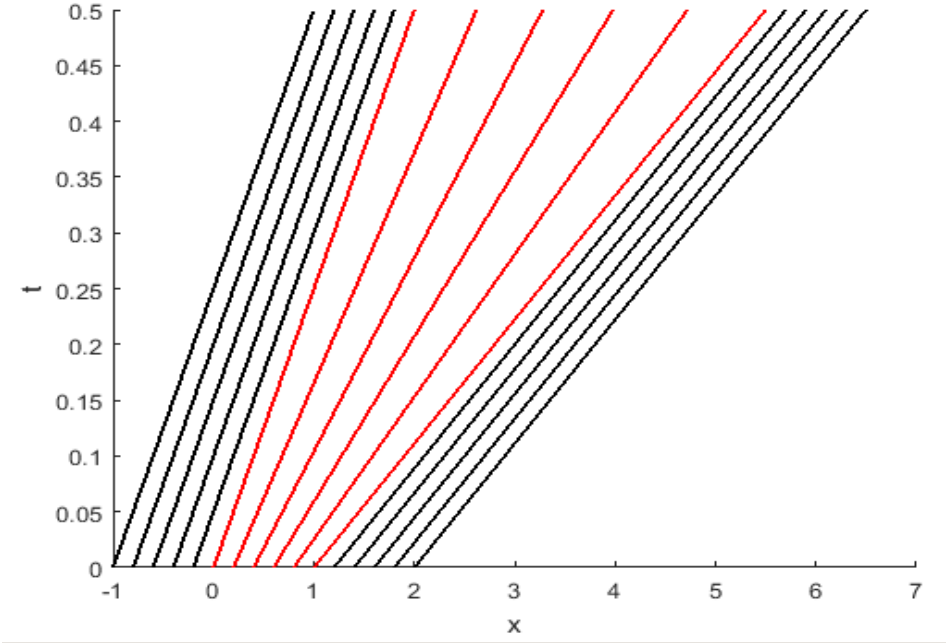
Prob 1

$$u(x,t) = \begin{cases} 1 & \text{, if } x < 4t \\ 1 + \dfrac{-(4t+1) + \sqrt{4tx + 8t + 1}}{2t} & \text{, if } 4t \le x \le 9t + 1 \\ 2 & \text{, if } x > 9t + 1 \end{cases}$$

Plot of solution:



Plot of characteristics:

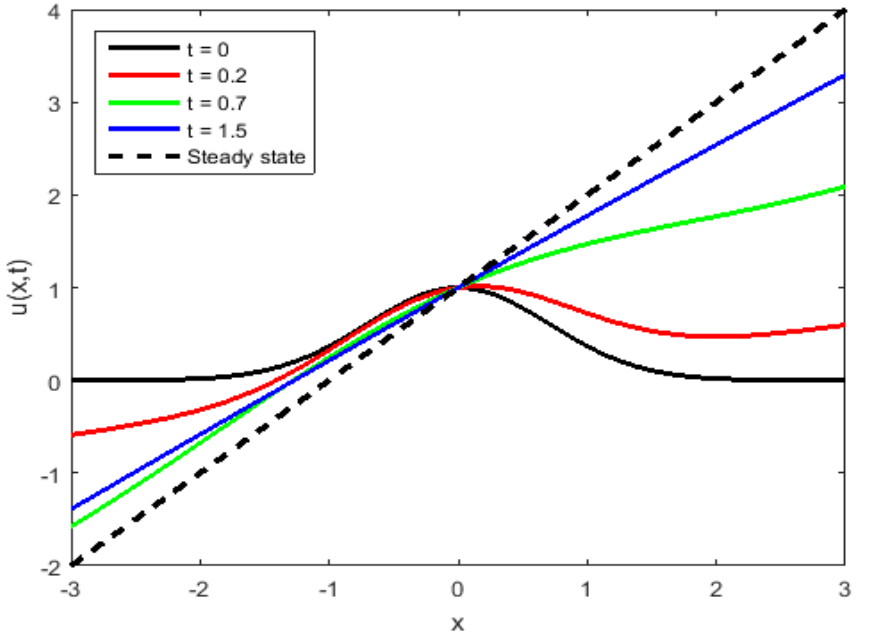


Prob 2

$$u(x,t) = e^{-[x e^{-(t+t^2/2)}]^2} + x\,[1 - e^{-(t + t^2/2)}]$$

Steady state solution is $u_s(x) = 1 + x$.

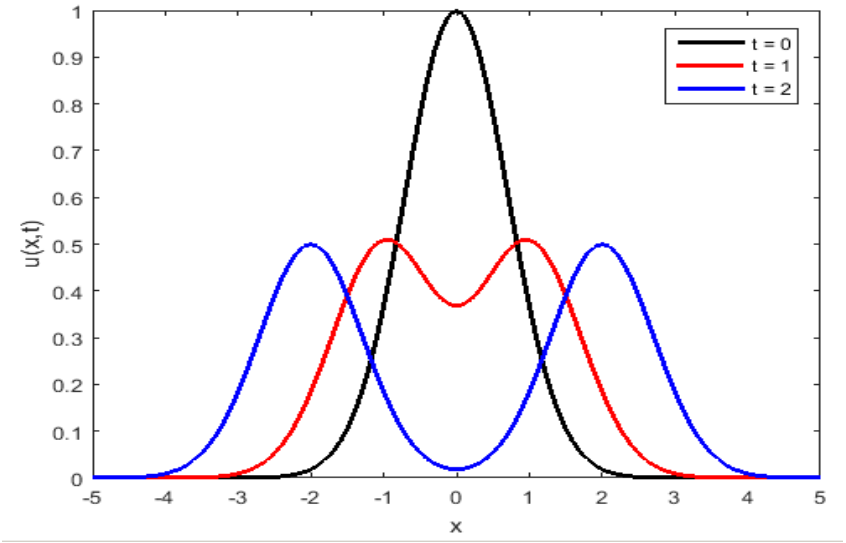
Plot of solution:



Prob 3

$$u(x,t) = \frac{1}{2}\,[e^{-(x-t)^2} + e^{-(x+t)^2}] \quad .$$

Plot of solution:



Prob 4

$$G(t,t') = \frac{1+t}{1+t'} \quad .$$