MAE/MSE 502, Fall 2017, Homework #5

<u>Three free points</u> for all who return this assignment on time with a properly filled cover sheet, and without a violation of the rules on collaboration for this homework. This brings the total points from homework assignments to 50.

Prob 1 (2 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE,

$$(1+t)\frac{\partial u}{\partial t} + x\,\frac{\partial u}{\partial x} = 1 \quad ,$$

with the boundary condition

$$u(x,0) = \exp(-x^2) \quad .$$

We expect a closed-form analytic solution for this problem. Plot the solution, $u(x,t)$, as a function of x at $t = 0$, 1, and 2. Make the plot over the range of $-5 \leq x \leq 5$. Please collect all 3 curves in one plot.

Prob 2 (2 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE,

$$\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x} = x \quad ,$$

with the boundary condition

$$u(x,0) = 0 \quad .$$

We expect a closed-form analytic solution for this problem. Plot the solution, $u(x,t)$, as a function of x at $t = 0$, 0.1, and 0.3. Make the plot over the range of $-3 \leq x \leq 3$. Please collect all 3 curves in one plot.

Prob 3 (3 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE,

$$\frac{\partial u}{\partial t} + (1-u)\,\frac{\partial u}{\partial x} = -u \quad ,$$

with the boundary condition

$$u(x,0) = \mathrm{P}(x) \, ,$$

where

$$\begin{aligned} \mathrm{P}(x) &= 0 \quad , \text{ if } x < 0 \\ &= x \quad , \text{ if } 0 \leq x \leq 1 \\ &= 1 \quad , \text{ if } x > 1 \, . \end{aligned}$$

We expect a closed-form analytic solution for this problem. Plot the solution, $u(x, t)$, as a function of x at $t = 0$, and 0.75, and 1.5. Make the plot over the range of $-1 \leq x \leq 3$. Please collect all 3 curves in one plot.