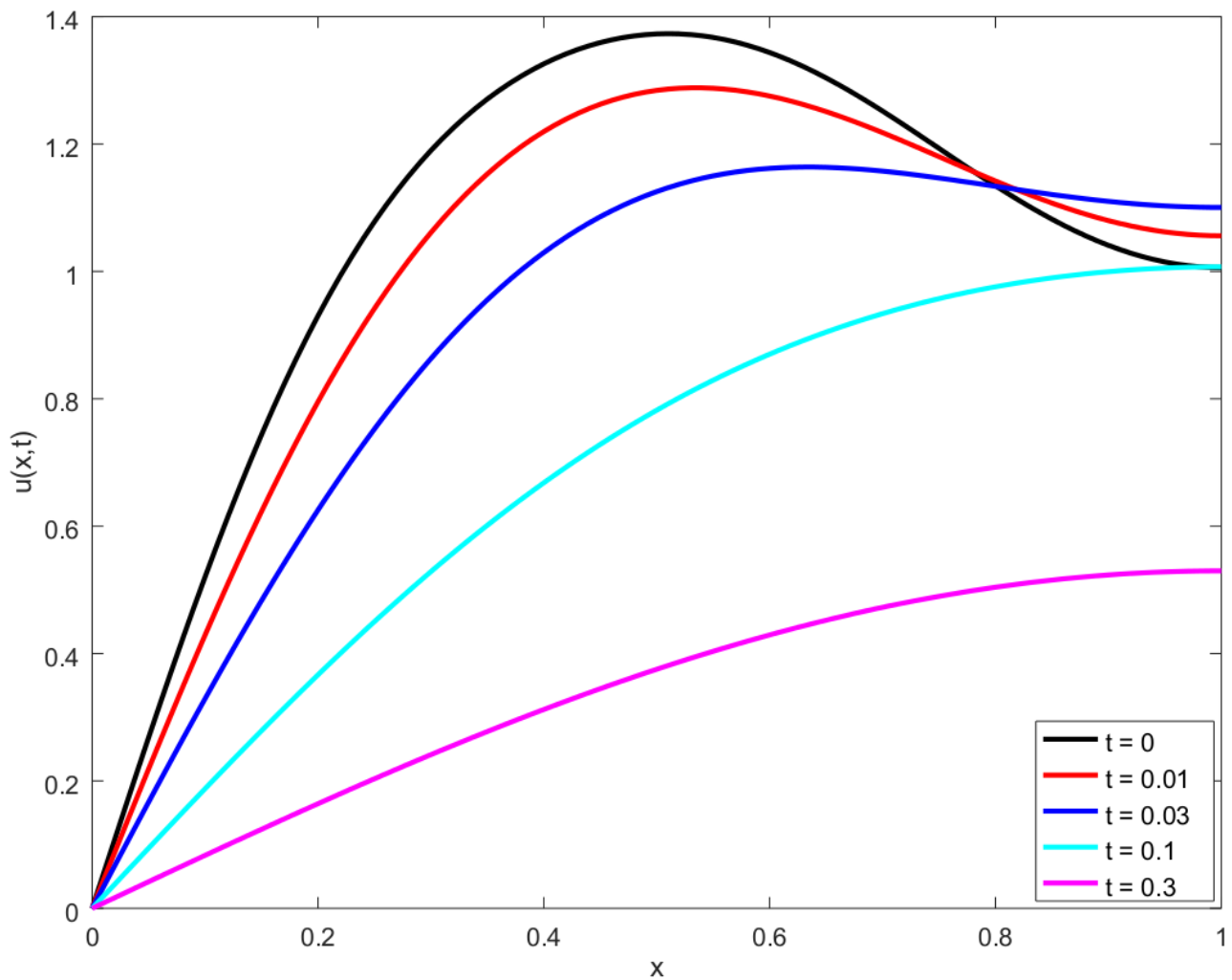MAE/MSE 502 (Spring 2017) HW1 Solutions

Prob 1

$$u(x,t) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{2}\right) \exp\left(-\left(\frac{n\pi}{2}\right)^2 t - \sin(t)\right)$$

where the summation is over <u>odd values of n only</u>.

$$a_n = \frac{\displaystyle\int_0^1 (2x^4 - 7x^2 + 6x)\sin\left(\frac{n\pi x}{2}\right) dx}{\displaystyle\int_0^1 \left[\sin\left(\frac{n\pi x}{2}\right)\right]^2 dx} \quad , \text{ if } n \text{ is odd.}$$
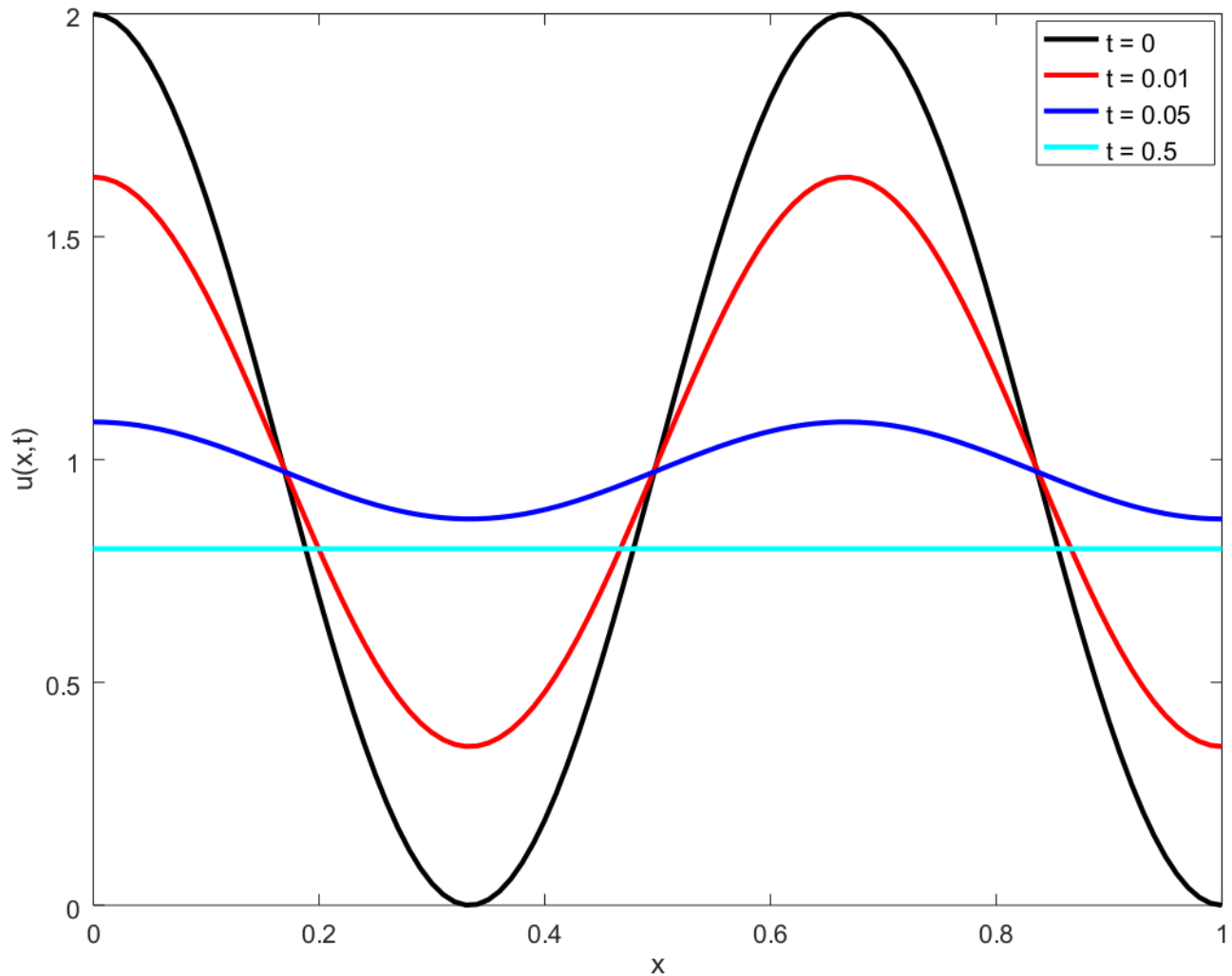
Plot (based on truncation at $n = 9$, inclusively):

Prob 2

$$u(x,t) = \frac{2}{2+t} + \left(\frac{2}{2+t}\right)^{9\pi^2+1} \cos(3\pi x)$$

Plot:

Prob 3

$$E(t) = \frac{1}{6(1+t)} + \frac{t+t^2/2}{t+1}$$

At $t = 1$, $E(t) = 5/6$. $E(t) \to \infty$ as $t \to \infty$. A steady state does not exist.

Prob 4

We will discuss the solution in class.