MAE/MSE 502, Spring 2017 Homework #2 solutions

Prob 1.

$$u(x,t) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{5}\right) \cos\left(\frac{n\pi t}{5}\right) \quad .$$

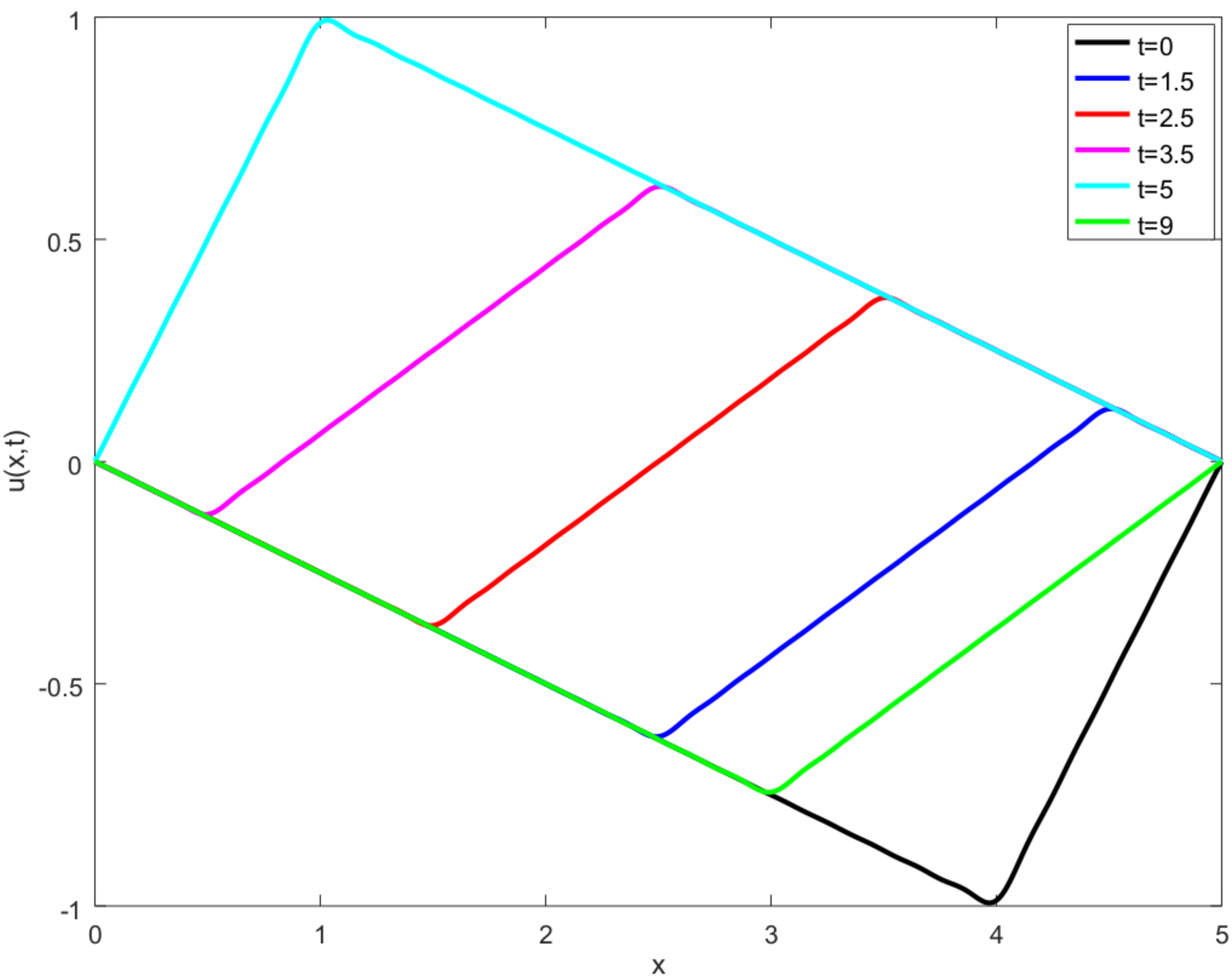
Case (a)

$$a_n = \frac{\int_0^4 -(x/4)\sin(n\pi x/5)\,dx + \int_4^5 (x-5)\sin(n\pi x/5)\,dx}{\int_0^5 [\sin(n\pi x/5)]^2\,dx} \quad .$$

Case (b)

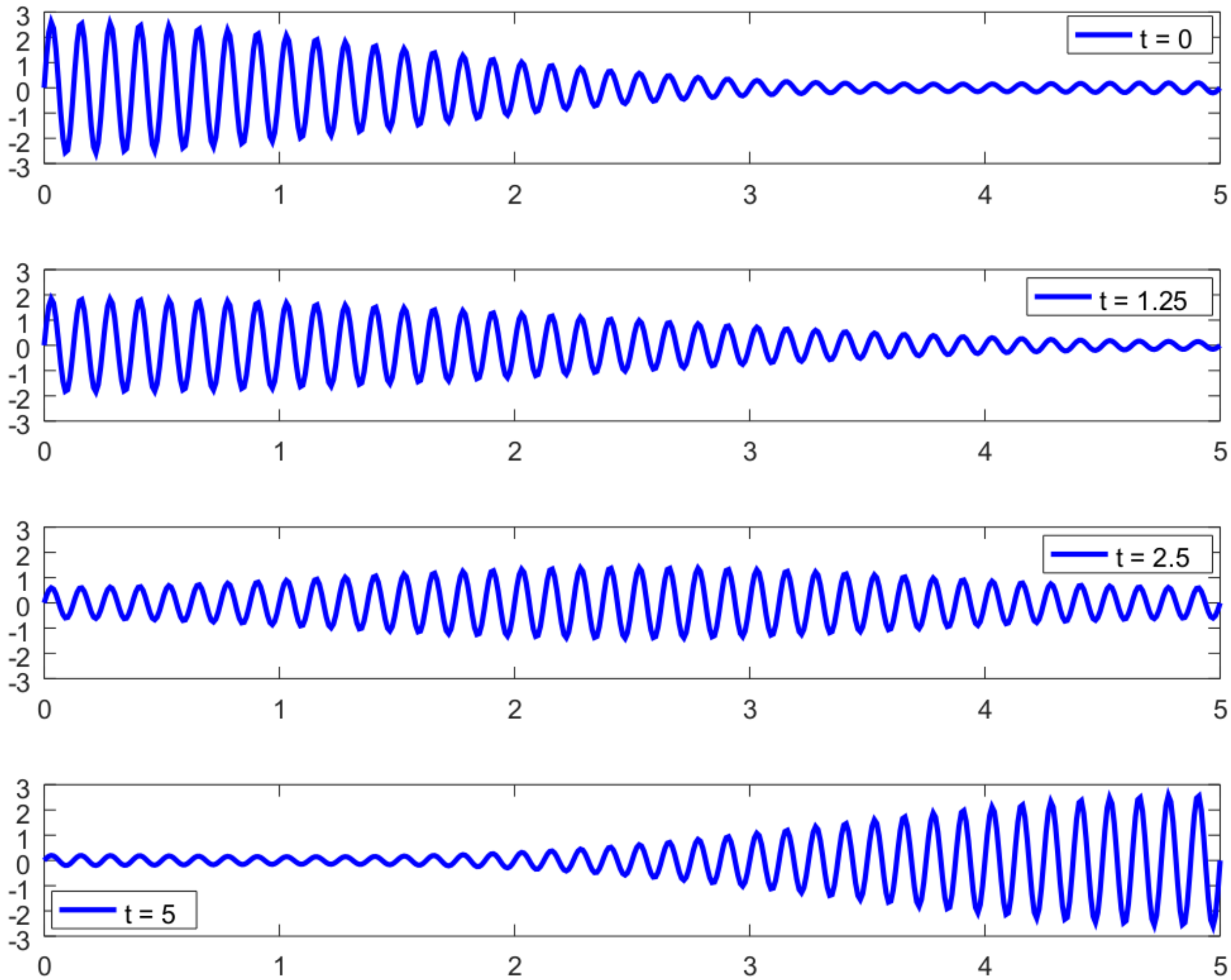
$a_{78}=0.2$, $a_{79}=0.6$, $a_{80}=1$, $a_{81}=0.6$, $a_{82}=0.2$ (all other coefficients are zero)

Plot for (a):



Plot for (b):

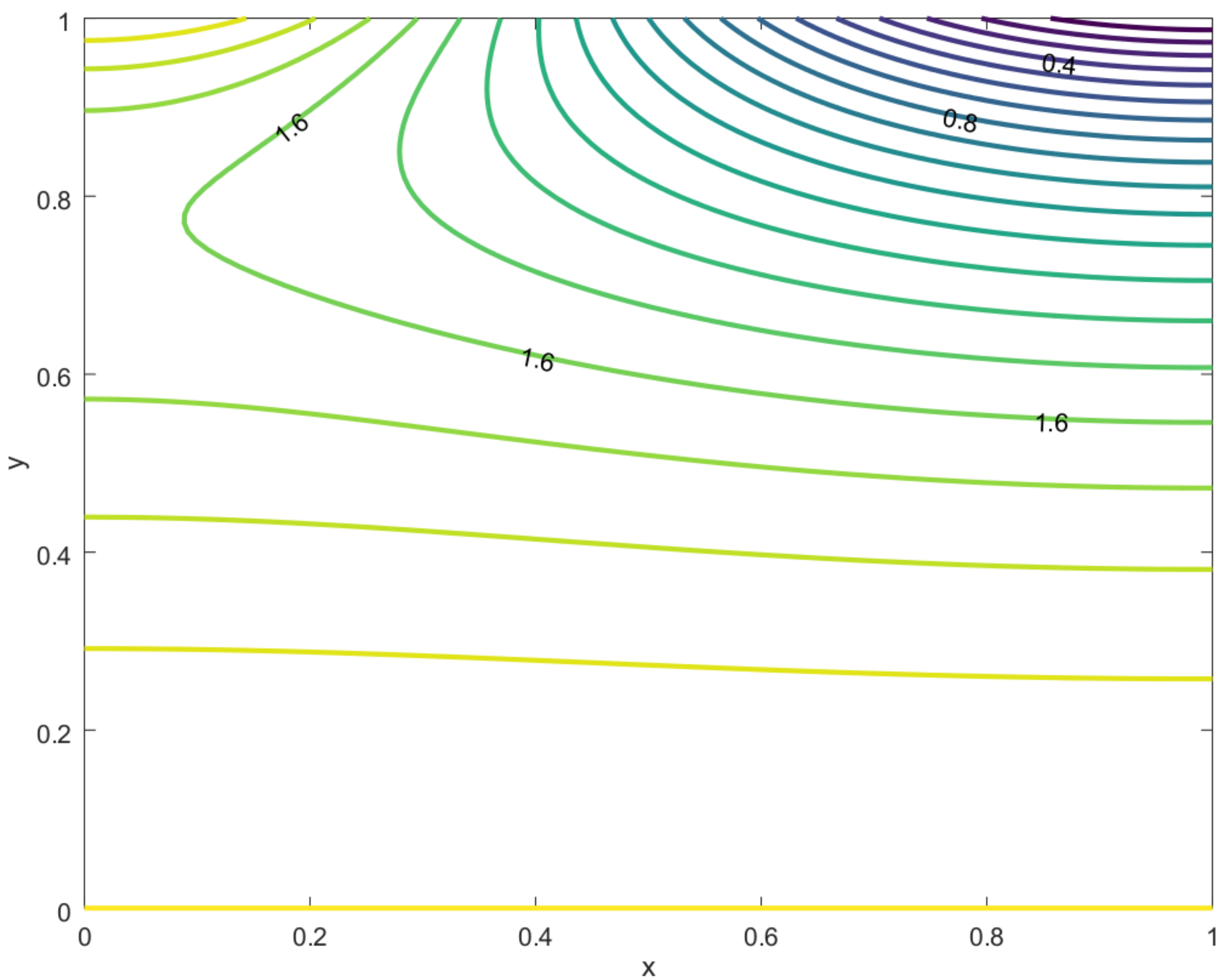
(abscissa is x, ordinate is $u(x,t)$)



Prob 2

$$u(x,y) = 2\cos(y) + \frac{1-2\cos(1)}{\sin(1)}\sin(y) + \frac{\sinh\left(\sqrt{4\pi^2-1}\,y\right)}{\sinh\left(\sqrt{4\pi^2-1}\right)}\cos(\pi x) \ .$$

Plot:



Prob 3

$E(t) \equiv 2/3$ for all t. $E(t) = 2/3$ as $t \to \infty$.

(Note: This is a somewhat unusual case that, even though $E(t)$ stays as a constant from the beginning, $u(x,y,t)$ still evolves with time. For example, one can readily verify, from the last b.c., that $\partial u/\partial t$ is not zero at $t = 0$. It is possible for the "total energy" to remain constant because the input from the boundary is balanced by the internal sink from the last term in the r.h.s. of the PDE.)

Prob 4

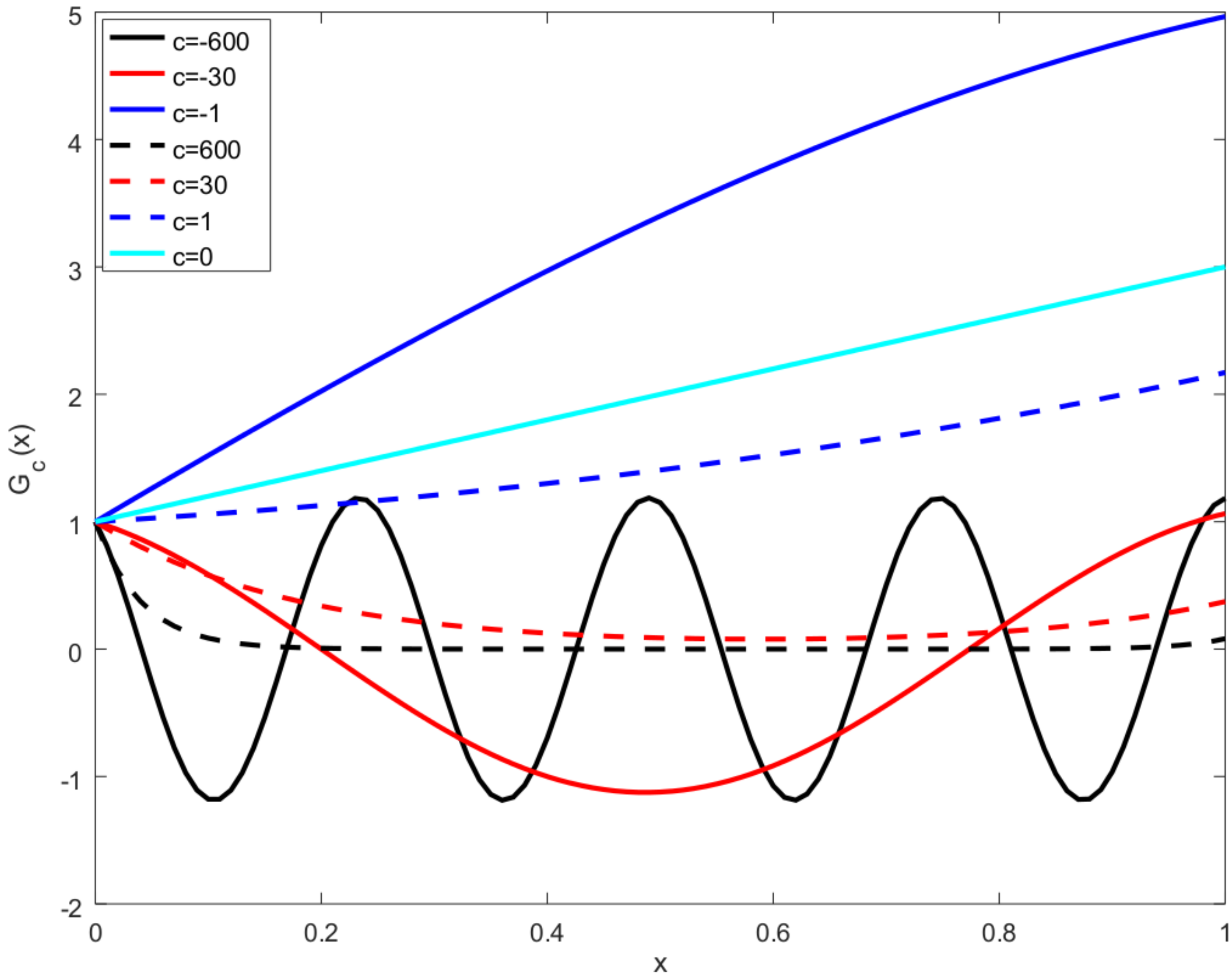
(a) All values of c (i.e., for $-\infty < c < \infty$) are eigenvalues, with the exceptions of isolated points at $c = -(n\pi/2)^2$, $n = 1, 3, 5, \ldots$ (At those points, a solution does not exist for the system.) The eigenfunctions are

$$G_c(x) = \cos\left(\sqrt{-c}\,x\right) + \frac{2 + \sqrt{-c}\,\sin\left(\sqrt{-c}\right)}{\sqrt{-c}\,\cos\left(\sqrt{-c}\right)}\sin\left(\sqrt{-c}\,x\right) \quad \text{, if } c < 0$$

$$G_c(x) = 2x + 1 \quad \text{, if } c = 0$$

$$G_c(x) = \cosh\left(\sqrt{c}\,x\right) + \frac{2 - \sqrt{c}\,\sinh\left(\sqrt{c}\right)}{\sqrt{c}\,\cosh\left(\sqrt{c}\right)}\sinh\left(\sqrt{c}\,x\right) \quad \text{, if } c > 0 \,.$$

(b) Plot:



(c) The orthogonality relation does not hold.

(d) No.