MAE/MSE 502, Spring 2017, Homework #5

One free point for all, which brings the total points from homework assignments to 50.

Prob 1 (3 points)

For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE,

$$\frac{\partial u}{\partial t} + \left(u^2 + u\right) \frac{\partial u}{\partial x} = 0 \quad ,$$

with the boundary condition

$$u(x,0) = \mathrm{P}(x) \ ,$$

where

$$\begin{aligned} \mathrm{P}(x) &= 0 \quad , \text{if } x < 0 \\ &= x \quad , \text{if } 0 \leq x \leq 1 \\ &= 1 \quad , \text{if } x > 1 \ . \end{aligned}$$

We expect a closed-form analytic solution for this problem. Plot the solution, $u(x, t)$, as a function of x at $t = 0$, and 0.5, and 1.

Prob 2 (3 points)

For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$\left(1+t\right) \frac{\partial u}{\partial t} + \left(1+x\right) \frac{\partial u}{\partial x} = \left(1+u\right) \quad ,$$

with the boundary condition

$$u(x,0) = \exp(-x^2)$$

We expect a closed-form analytic solution for this problem. Plot the solution, $u(x,t)$, as a function of x at $t = 0$, 0.2, and 0.5. Please collect all 3 curves in one plot.

Prob 3 (3 points)

For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} \quad ,$$

with the boundary conditions:

(i) $u(x, 0) = x$

(ii) $u_t(x,0) = x^2$.

We expect a closed-form analytic solution for this problem. Plot the solution, $u(x,t)$, as a function of x at $t = 0$, 0.5, and 1. Please collect all 3 curves in one plot.