STOP

Please read the rules before forming any collaboration for the homework. A violation of the rule(s) given in this page will be considered a violation of ASU's Academic Integrity Policy.

Rules on collaboration for homework:

(1) Collaboration is not allowed unless all involved follow rules (2)-(3) and unless the extent of collaboration is properly disclosed in a statement in the first page of the report for the homework. An additional instruction will be given in Homework #1 on the proper format of the statement.

(2) For each homework assignment, each person can have maximum of one collaborator. Be aware that a collaborator's collaborator counts as a collaborator. For example, if Alice collaborates with Bob and Bob collaborates with Charles, Charles counts as a collaborator of Alice. All three violate the rule. In other words, collaboration can only be carried out within an isolated "team of two". Please talk to a potential collaborator to ensure that this rule is not violated before establishing any collaboration.

(3) In a legitimate collaboration, each individual must make a non-negligible contribution to the collaborative effort. Taking the solution or computer code from another student without making a reciprocal contribution to it is not legitimate collaboration and is not allowed. To certify that a collaboration is legitimate, the contribution from each person to the collaborative effort must be documented in the mandatory statement as described in (1).

MAE/MSE 502, Fall 2018 Homework #1
1 point ≈ 1% of your total score for this class

<u>Hard copy</u> of report is due 6:00 PM on the due date. Please see the cover page of this document for the rules on collaboration for homework. Please follow the rules. <u>Computer codes used to complete the tasks should be included in the report.</u>

Task 0 (no point, but mandatory to complete for the report to be accepted)
Provide a statement to address whether collaboration occurred in completing this assignment. **This statement must be placed in the beginning of the first page of the report.** If no collaboration occurred, simply state "No collaboration". This implies that the person submitting the report has not helped anyone or received help from anyone on this assignment. If collaboration occurred, provide the name of collaborator (only one allowed), a list of the task(s) on which collaboration occurred, and descriptions of the extent of collaboration. For example:

Name of collaborator: Joe Smith	
Task(s), specific detail	Contribution to collaborative effort
Task 1, Matlab code	Discussed strategy for coding and checked errors in the code with collaborator
Task 3, Mathematical derivation	Worked with collaborator to derive the formula for E(t)

Task 1 (4 points)
(a) For $u(x, t)$ defined on the domain of $0 \leq x \leq 1$ and $t \geq 0$, solve the PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \ ,$$

with the boundary conditions,

(i) $u_x(0, t) = 0$, (ii) $u_x(1, t) = 0$, and   (iii) $u(x, 0) = P(x)$,

where

$$P(x) = \frac{[1 - \cos(2\pi x)]^8}{256} + \frac{1 - \cos(\pi x)}{4}$$ (Note the 8th power in the first term)

Plot the solution, $u(x,t)$, as a function of x at $t = 0$, 0.005, 0.03, and 0.1. Please collect all four curves in a single plot.

(b) Repeat (a) but now solve the system with the first two boundary conditions changed to

(i) $u(0, t) = 0$, (ii) $u_x(1, t) = 0$.

The 3rd boundary condition remains the same. Plot the solution, $u(x,t)$, as a function of x at $t = 0$, 0.005, 0.03, and 0.1. Please collect all four curves in a single plot.

Note for Task 1:
(i) We expect the solution to be expressed as an infinite series. It is part of your job to determine the appropriate number of terms to keep in the series to ensure that the solution is accurate. As a useful measure, the solution at $t = 0$ should nearly match the given initial state in the 3rd boundary condition. If they do not match, either the solution is wrong or more terms need to be retained in the series. This remark applies to all future homework problems that require the evaluation of an infinite series.
(ii) For the evaluation of the expansion coefficients in the infinite series, there is no need to carry out the integrations analytically. (If you wish to do so, please feel free to use an online integrator or similar software such as Wolfram Alpha. No need to do it by hand.) It is perfectly acceptable (actually, recommended for this task) to evaluate the integrals numerically using Matlab or its equivalent. See additional note in the last page for an example of using Matlab to numerically evaluate an integral.

Task 2 (2 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 1$ and $t \ge 0$, solve the PDE,

$$\frac{\partial u}{\partial t} = (1+t)^2 \frac{\partial^2 u}{\partial x^2} - 3\,u \ ,$$

with the boundary conditions,

(i) $u(0, t) = 0$, (ii) $u(1, t) = 0$, and (iii) $u(x,0) = 2\sin(\pi x) + \sin(3\pi x)$.

We expect a closed-form analytic solution with a finite number of terms and without any unevaluated integral. A deduction will be assessed on any unevaluated integral that is left in the final answer. Plot the solution, $u(x, t)$, as a function of x at $t = 0, 0.003, 0.01, 0.03,$ and 0.1. Please collect all curves in one plot.

Task 3 (1.5 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 1$ and $t \ge 0$, consider the PDE,

$$\frac{\partial u}{\partial t} = e^{-t} \frac{\partial^2 u}{\partial x^2} - 3\,u \ ,$$

with the boundary conditions,

(i) $u_x(0, t) = 1$, (ii) $u_x(1, t) = 2$, and (iii) $u(x,0) = 0.5\,x^2 + x$

If the "total energy" of the system is defined as

$$E(t) \equiv \int_0^1 u(x,t)\,dx \ ,$$

find the exact (analytic) expression of $E(t)$. Plot $E(t)$ as a function of t over the range of $0 \le t \le 1$. (Note: The only deliverable of this task is $E(t)$. You may or may not need to find the full solution of the PDE to answer the key question.)

Task 4 (0.5 point)

For the heat transfer problem, the Heat equation in its dimensional form is

$$\frac{\partial u}{\partial \hat{t}} = K \frac{\partial^2 u}{\partial \hat{x}^2} \;, \quad 0 \le \hat{x} \le L \;\; (L \text{ is the length of the "metal rod", in meters}) \text{ and } \hat{t} \ge 0 \;, \quad (1)$$

where $\hat{t}$ and $\hat{x}$ are time in seconds and distance in meters, K is thermal diffusivity in m²/s, and u is temperature. In our class, we usually consider the non-dimensionalized version of (1),

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \;, \quad 0 \le x \le 1 \text{ and } t \ge 0 \;, \tag{2}$$

where x is related to $\hat{x}$ by $\hat{x} = L\,x$. In (2), time is also non-dimensionalized by $\hat{t} = T\,t$, where T is a certain dimensional time scale, and so on.

(a) In order to claim that the non-dimensionalized system, (1), is equivalent to its dimensional counterpart, (2), the three parameters K, L, and T must satisfy a unique relation. First, find out what this relation is. (For the discussion in part (b) & (c), it is useful to write the relation as $T = f(K, L)$.)

(b) Suppose that the non-dimensionalized Heat equation, (2), is used to model the real world problem of heat transfer along a metal rod that is 1 meter long and made of copper ($K \approx 0.0001$ m²/s), what would be the actual time, in seconds, that $t = 0.01$ corresponds to in that problem?

(c) Same as (b), but suppose that (2) describes heat transfer along a wooden stick that is 0.3 meter long and made of pine wood ($K \approx 10^{-7}$ m²/s), what would be the actual time, in seconds, that $t = 0.01$ corresponds to? (We consider $t = 0.01$ because it is about the time when a significant redistribution of temperature begins to take place in the scenario described in Part (d).)

(d) Are the time scales you obtained in (b) and (c) consistent with daily experience? For instance, one can use a long wooden spoon to continuously stir a boiling pot of soup without getting one's hand burned. In contrast, the same practice would make one very uncomfortable if the spoon is made entirely of copper. Note that the time scale for cooking a pot of soup is about 10 minutes. The length of a big wooden spoon is about a foot, or 0.3 meter.

Additional note: Using Matlab to numerically evaluate an integral

One of the simplest Matlab functions for this purpose is **trapz**. It uses the trapezoidal method to evaluate an integral. For example, to numerically evaluate

$$I = \int_0^1 \sin(x)\, dx$$

with $\Delta x = 0.01$, we first construct the discretized arrays of the coordinate points and the values of the integrand at those points. We then call **trapz** with those two arrays as the input to complete the integration. The Matlab code is very simple:

```
x = [0:0.01:1];
y = sin(x);
Integ = trapz(x,y)
```

One can readily verify the outcome with the analytic result of $I = 1 - \cos(1)$.

When using "trapz" or similar functions for numerical integration, it is important to use a sufficient grid resolution to avoid "aliasing errors". It requires a minimum of 5 points (10 points are recommended) to resolve one cycle of a sinusoidal wave. In the example above, 100 grid points are used to resolve less than 1/6 of a cycle, which is sufficient. If the integrand in the above example is changed to y = sin(500 x), the result of numerical integration will be entirely wrong. (The remedy would be to also increase the grid resolution. For example, change the grid to x = [0:0.0001:1].)