MAE Fall 2018 HW1 Solutions

Task 1

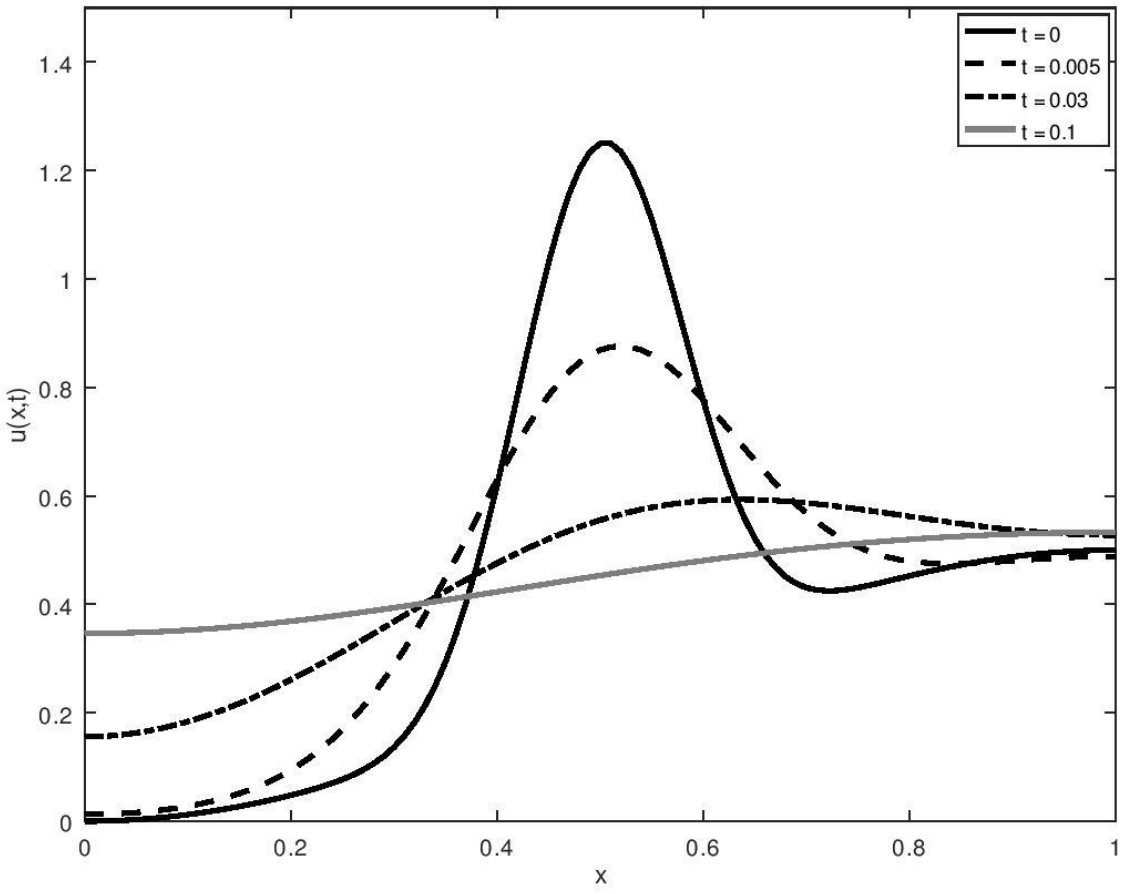
(a)

$$u(x,t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\pi x) \exp(-(n\pi)^2 t)$$

where

$a_0 = \int_0^1 P(x)\, dx$, and $a_n = 2\int_0^1 P(x) \cos(n\pi x) dx$ for $n > 0$.

Plot:



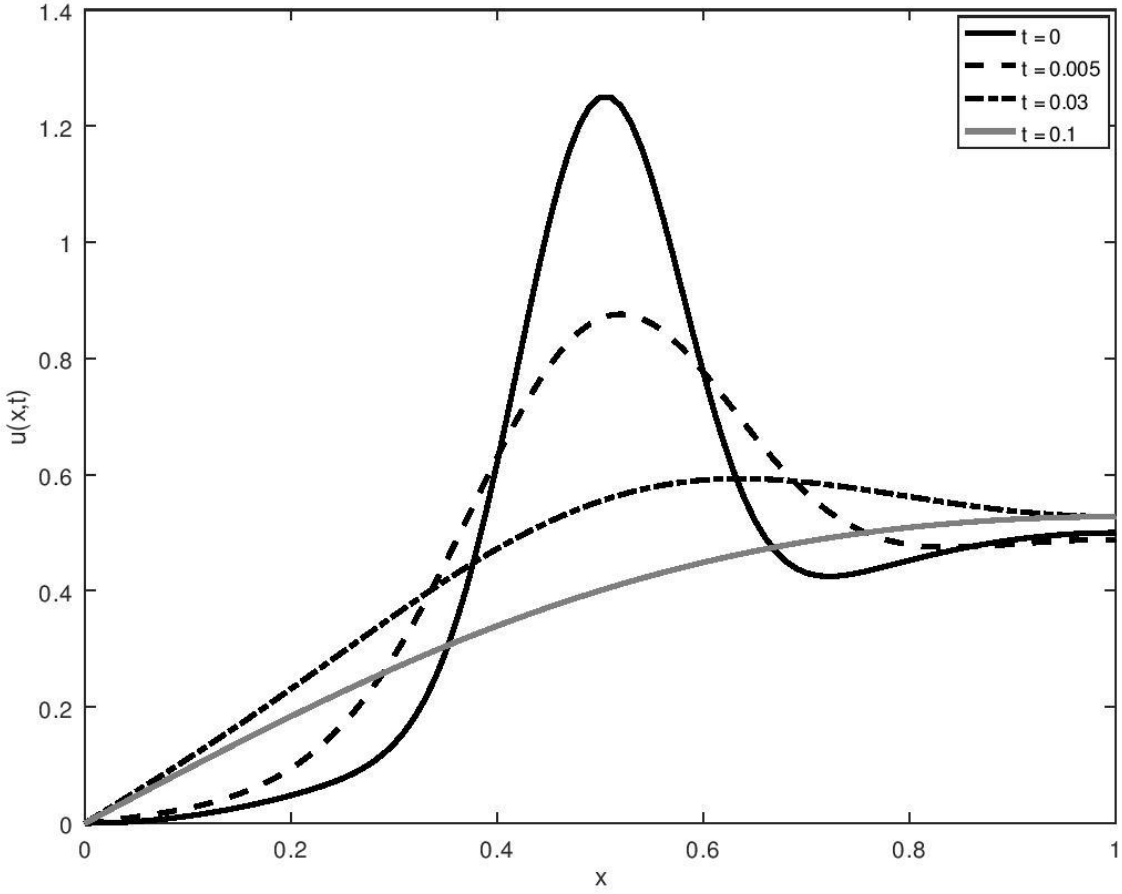
(b)

$$u(x,t) = \sum_{n=1}^{\infty} a_n \sin(\frac{n\pi x}{2}) \exp(-(\frac{n\pi}{2})^2 t)$$

where the summation is over <u>odd values of *n* only</u>, and

$a_n = 2\int_0^1 P(x)\,\sin(\frac{n\pi x}{2})dx$, for odd values of n.

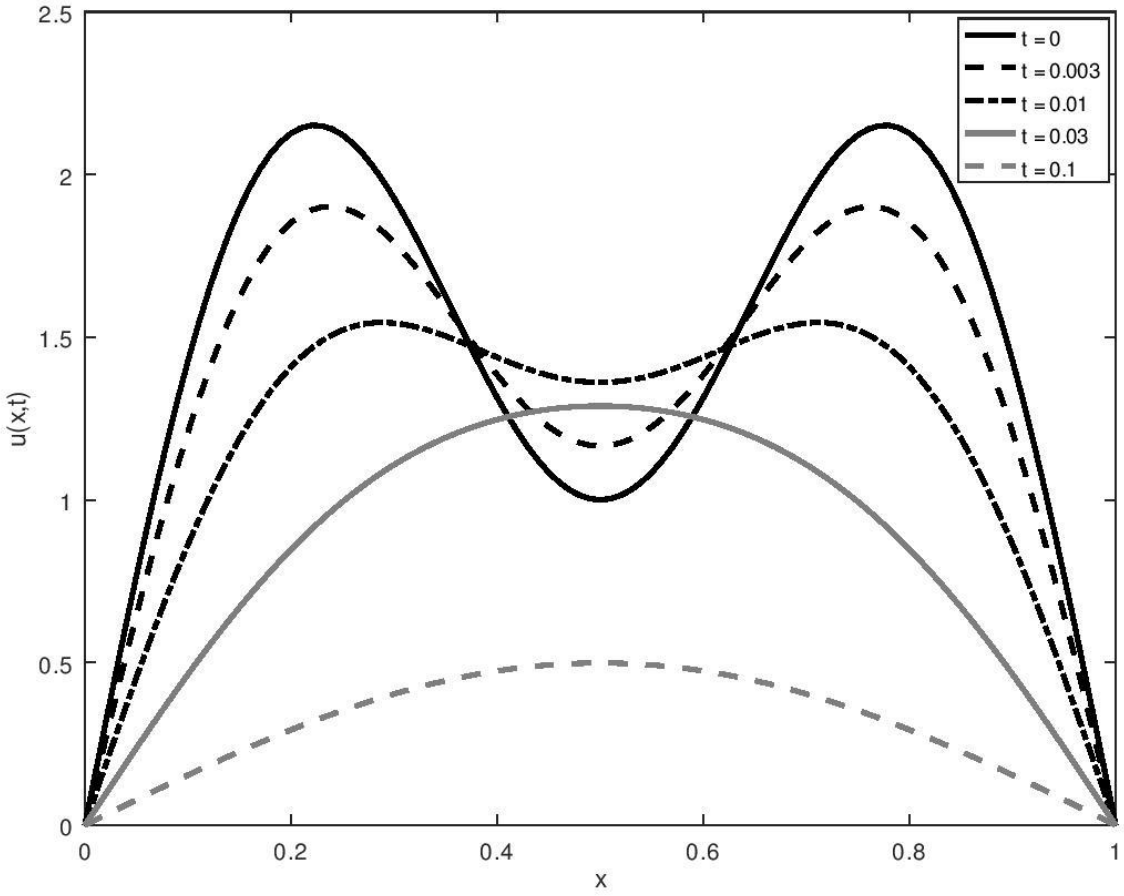
Plot:



Task 2

$$u(x,t) = 2\, e^{-\pi^2\left(t+\, t^2+\frac{1}{3}t^3\right)-3t} \sin(\pi x) +\, e^{-9\pi^2\left(t+\, t^2+\frac{1}{3}t^3\right)-3t} \sin(3\pi x)$$

Plot:



Task 3

$$E(t) = \frac{1}{6}\, e^{-3t} + \frac{1}{2}\, e^{-t}$$

Task 4

We will discuss the solution of this task in class.