MAE Fall 2018 HW2 Solution

Task 1

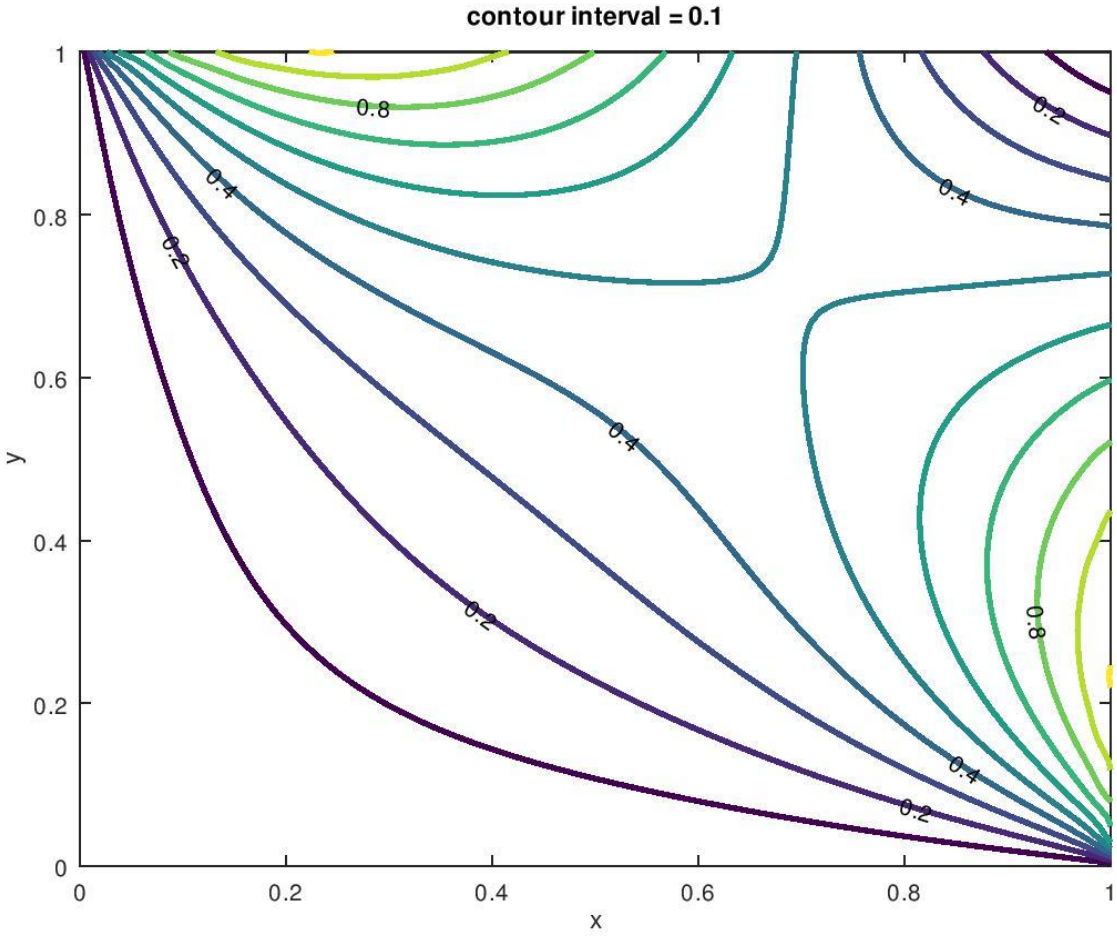
$$u(x,y) = \sum_{n=1}^{\infty} a_n \sin(n\pi y)\sinh(n\pi x) + b_n \sin(n\pi x)\sinh(n\pi y)$$

where

$$a_n = \frac{2}{\sinh(n\pi)} \int_0^1 4\left(\sqrt{y} - y\right)\sin(n\pi y)\, dy$$

$$b_n = \frac{2}{\sinh(n\pi)} \int_0^1 \sin(\pi\sqrt{x})\sin(n\pi x)\, dx$$

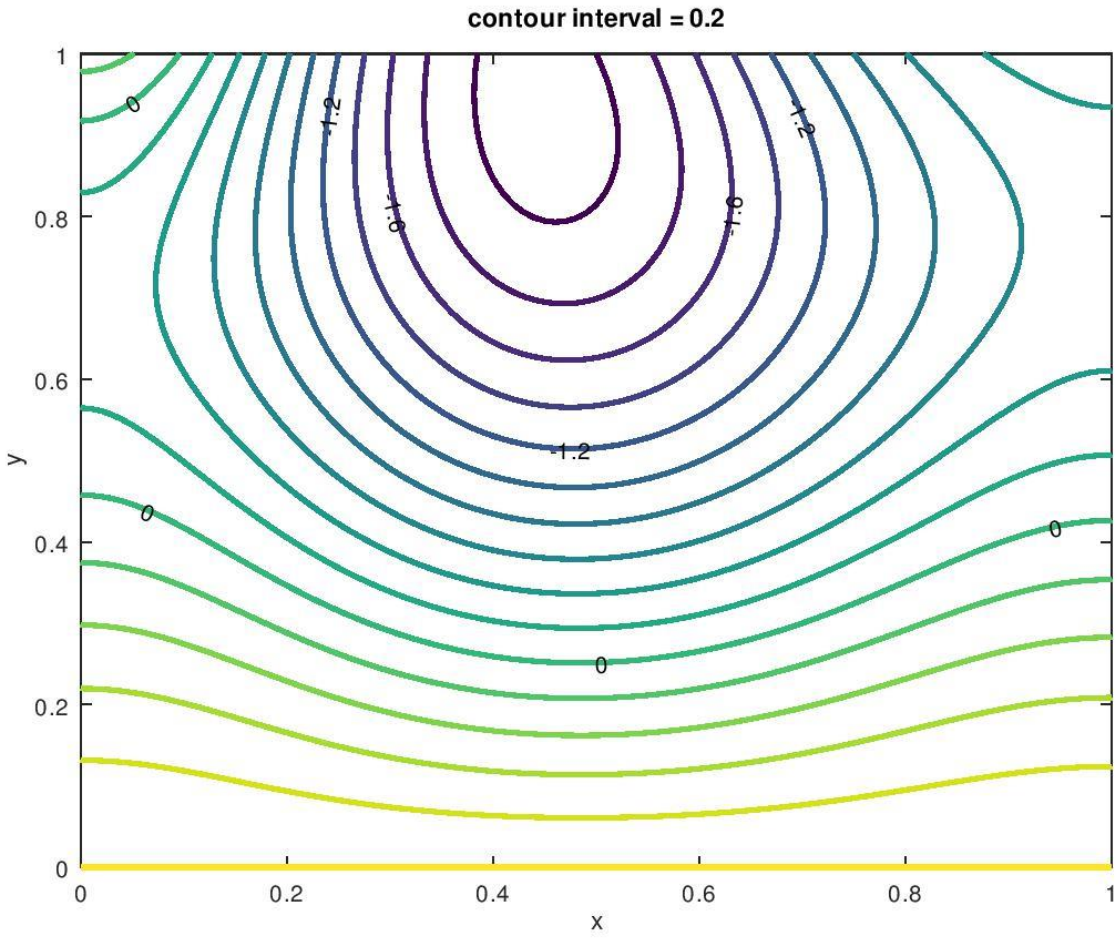
Plot:



Task 2

$$u(x,y) = \cos(\pi y) - \frac{2}{\pi}\sin(\pi y) + y\cos(2\pi x) + \frac{\sinh\left(\frac{\sqrt{5}}{2}\pi y\right)\cos(3\pi x)}{\left(\frac{\sqrt{5}}{2}\pi\right)\cosh\left(\frac{\sqrt{5}}{2}\pi\right)}$$

Plot:



Task 3

(a) There are multiple solutions.

(b)

$$u(x,y) = B + y + \frac{\cosh(\pi y)\cos(\pi x)}{\pi\sinh(\pi)}$$

where B is an arbitrary constant.

(c) There is no solution.

Task 4

$$K = -\frac{5}{6}$$

Task 5

$$u(x,t) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{6}\right)\cos\left(\frac{n\pi t}{6}\right)$$

where

$$a_n = \frac{1}{3}\left[\int_0^2 x \sin\left(\frac{n\pi x}{6}\right) dx + \int_2^6 \left(\frac{6-x}{2}\right) \sin\left(\frac{n\pi x}{6}\right) dx\right]$$

Plot: