MAE502 Fall 2018 HW3 Solution

Task 1

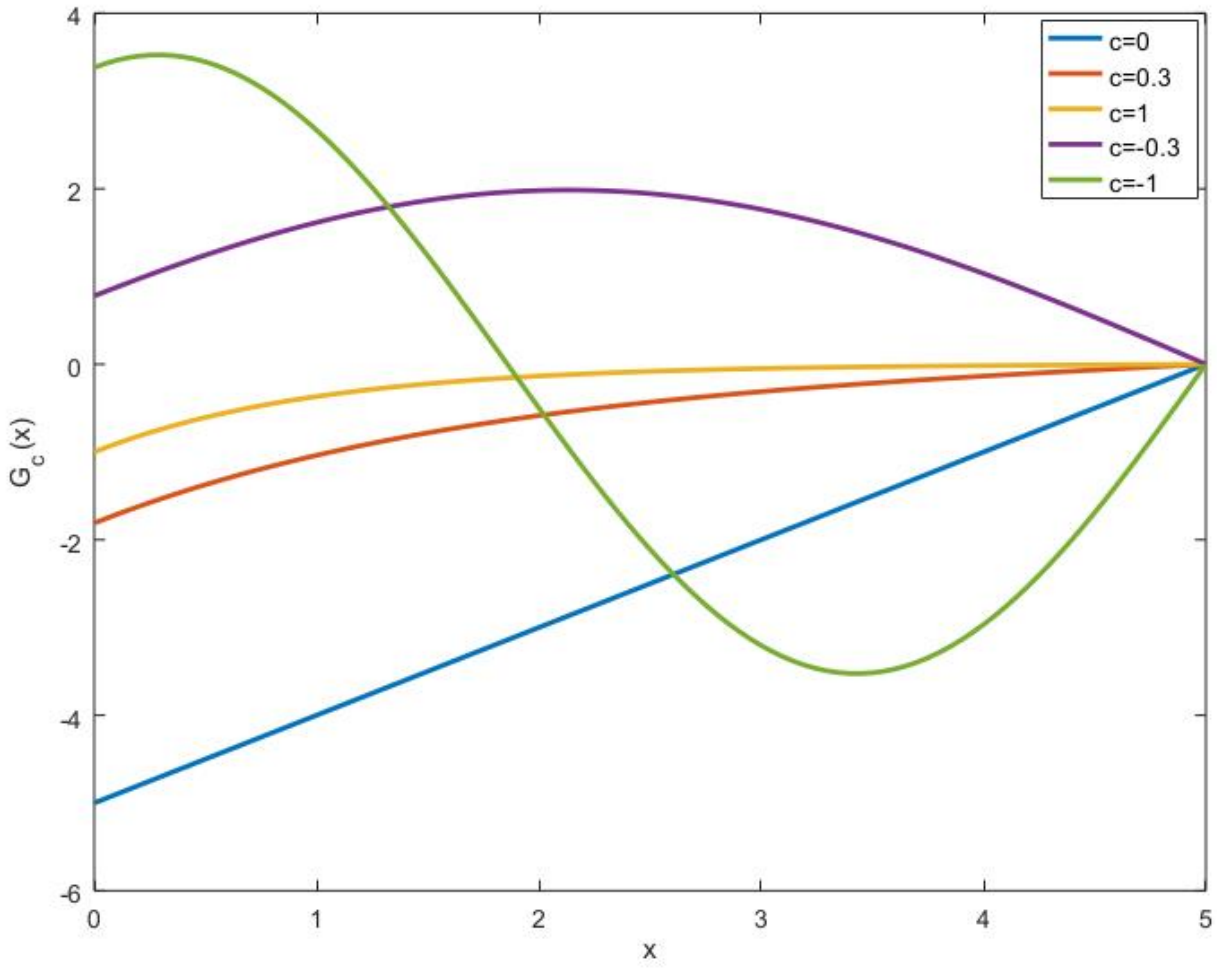
(a) All values of c on the real line, $-\infty < c < \infty$, are eigenvalues, except isolated points at $c = -\left(\frac{n\pi}{10}\right)^2$ with n = 1, 3, 5, …, which should be excluded. The eigenfunctions are

$$G_c(x) = \frac{\sinh((x-5)\sqrt{c}\,)}{\sqrt{c}\,\cosh(5\sqrt{c}\,)} \quad , \quad if\ c > 0$$

$$G_c(x) = x - 5 \quad , \quad if\ c = 0$$

$$G_c(x) = \frac{\sin((x-5)\sqrt{-c}\,)}{\sqrt{-c}\,\cos(5\sqrt{-c}\,)} \quad , \quad if\ c < 0\ and\ c \neq -\left(\frac{n\pi}{10}\right)^2, n = 1, 3, 5, \ldots$$

(b) Plot:



(c) The orthogonality relation does not hold.

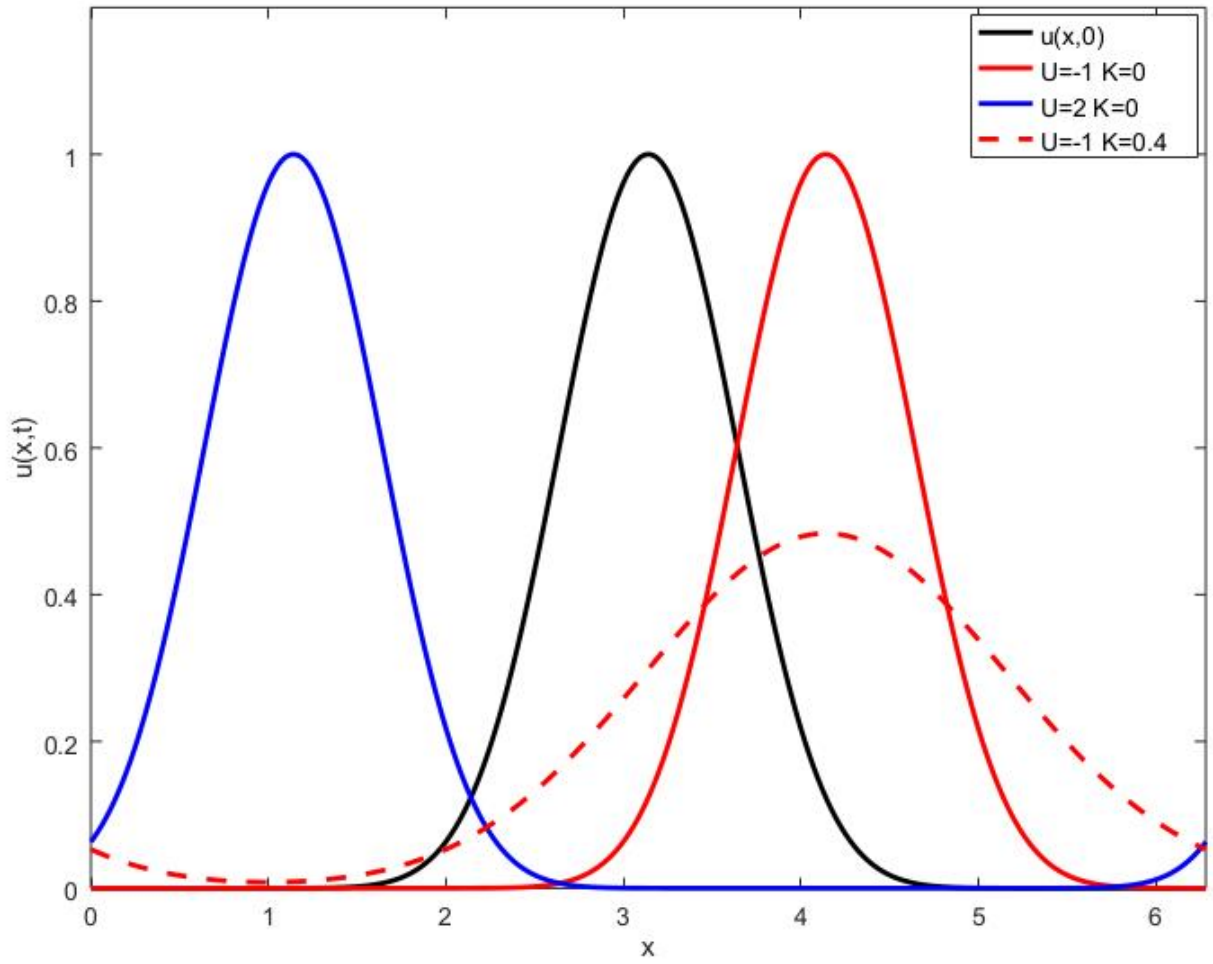
Task 2

$$u(x,t) = \sum_{n=-\infty}^{\infty} C_n(0)\, e^{(i\,n\,U - n^2 K)t} e^{i\,n\,x} \ ,$$

where

$$C_n(0) = \frac{1}{2\pi} \int_0^{2\pi} \frac{[1-\cos(x)]^8}{256}\, e^{-i\,n\,x}\, dx \ .$$

Plot:



Task 3

$$u(x,t) = (1+t)^2 + \cos(x + \ln(1+t))$$

Task 4

$$u(x,t) = \cos(2t) + \frac{3}{2}\sin(2t) + 4\,t\cos(x) + 2\sin(x)$$