MAE502 Fall 2018 HW4 Solution

Task 1

$$u(x,t) = \frac{e^{-t} + 3e^{-3t}}{2} + \frac{e^{-t} + (1+\pi^2)e^{-(3+\pi^2)\,t}}{(2+\pi^2)}\cos(\pi x)$$

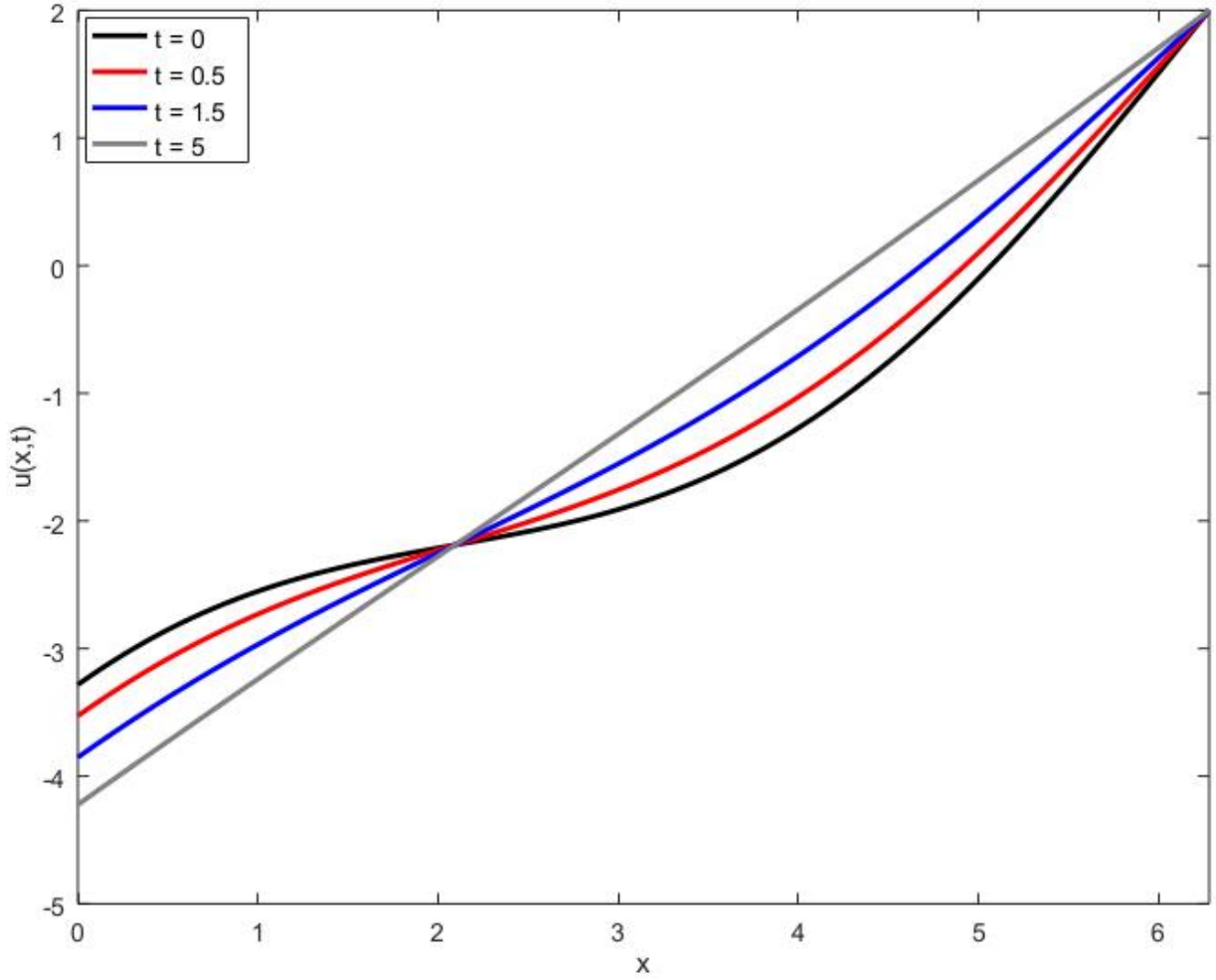
Task 2

$$u(x,t) = \frac{t^3}{6} + \frac{1-\cos(\sqrt{3}\,t)}{3}\cos(x) + [t - \sin(t)]\,\sin(2x)$$

Task 3

$$u(x,t) = x + 2 - 2\pi + \cos\left(\frac{3x}{4}\right)\, e^{-\frac{9}{16}t}$$

Plot:



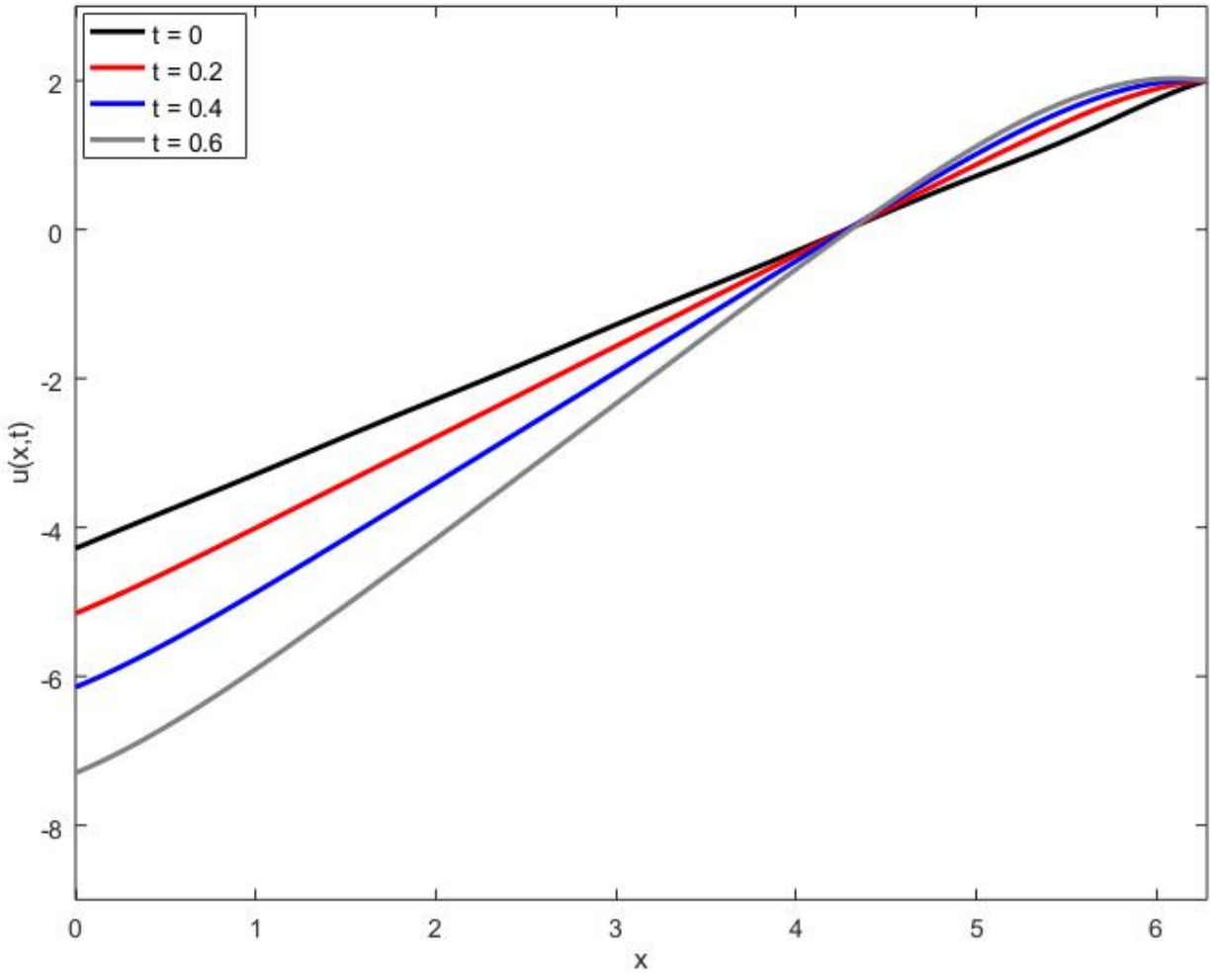
Task 4

$$u(x,t) = 2\cos(x) + \sin(x) + \sum_{\substack{n=1 \\ \{n\ odd\}}}^{\infty} a_n \cos\left(\frac{nx}{4}\right) e^{\left(1-\frac{n^2}{16}\right)t}$$

where the summation is over odd values of *n* only, and

$$a_n = \frac{1}{\pi}\int_0^{2\pi} [x + 2 - 2\pi - 2\cos(x) - \sin(x)] \cos\left(\frac{nx}{4}\right) dx$$

Plot (see an example of Matlab code in next page):



For Task 4, the full solution does not approach the steady state as $t \to \infty$. This is because the steady solution is unstable.

Code for Task 4

```
clear
dx = 2*pi/300; x = [0:dx:2*pi];
p = (x+2-2*pi)-(2*cos(x)+sin(x));
N = 15; t = [0 0.2 0.4 0.6];
for n = 1:N
  a(n) = trapz(x,p.*cos(n*x/4))/trapz(x,cos(n*x/4).^2);
end
for it = 1:length(t)
  u(it,:) = 2*cos(x)+sin(x);
  for n = 1:N
    if (mod(n,2) == 1)
      u(it,:) = u(it,:)+a(n)*cos(n*x/4)*exp((1-(n/4)^2)*t(it));
    end
  end
end
plot(x,u(1,:),'k-',x,u(2,:),'r-',x,u(3,:),'b-',...
     x,u(4,:),'-','Color',[0.5 0.5 0.5],'LineWidth',2)
xlabel('x');ylabel('u(x,t)');
legend('t = 0','t = 0.2','t = 0.4','t = 0.6','Location','NorthWest')
axis([0 2*pi -9 3])
```