# MAE502, Fall 2018, Homework #4

Hard copy of report is due 6:00 PM on the due date. Computer codes used to complete the tasks should be included in the report.

**Task 0** (no point, but mandatory to complete for the report to be accepted)
Provide a statement to address whether collaboration occurred in completing this assignment. **This statement must be placed in the beginning of the first page of report.** See related clarifications in Homework #1.

**Task 1** (2 points)
For $u(x, t)$ defined on the domain of $0 \le x \le 1$ and $t \ge 0$, solve the PDE,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - 3u + [1 + \cos(\pi x)]\exp(-t)$$

with the boundary conditions,

(i) $u_x(0, t) = 0$ , (ii) $u_x(1, t) = 0$ , (iii) $u(x, 0) = 2 + \cos(\pi\, x)$

We expect a closed-form solution with only a finite number of terms and without any unevaluated integrals.

**Task 2** (3 points)
For $u(x, t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, solve the PDE,

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2} + \frac{\partial^4 u}{\partial x^4} + t + \cos(x) + \sin(2x)\sin(t)$$

with periodic boundary conditions in $x$-direction, and the boundary conditions in the $t$-direction given as

(i) $u(x, 0) = 0$ , (ii) $u_t(x, 0) = 0$ .

We expect a closed form solution with only a finite number of terms and without any unevaluated integrals.

**Task 3** (1.5 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, solve the PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

with the boundary conditions,

(i) $u_x(0, t) = 1$ (ii) $u(2\pi, t) = 2$ (iii) $u(x, 0) = x + 2 - 2\pi + \cos(0.75\, x)$

We expect a closed form solution with only a finite number of terms and without any unevaluated integrals. Plot the solution, $u(x, t)$, as a function of $x$ at $t = 0$, 0.5, 1.5, and 5. Does the full solution approach the steady solution as $t \to \infty$? [Note: This system is the same as that given in Part (a) of Problem 2 in midterm exam, except for a slight modification of the 3rd boundary condition which does not alter the steady solution.]

**Task 4** (2.5 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, solve the PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u$$

with the boundary conditions,

(i) $u_x(0,t) = 1$ (ii) $u(2\pi,t) = 2$ (iii) $u(x,0) = x + 2 - 2\pi$ ($u_x$ is $\partial u/\partial x$) .

For this problem, it is acceptable to express the solution as an infinite series. Plot the solution, $u(x, t)$, as a function of $x$ at $t = 0$, 0.2, 0.4, and 0.6. Does the full solution approach the steady solution as $t \to \infty$? [Note: This system is the same as that given in Part (b) of Problem 2 in midterm exam.]

Explanatory note concerning the steady solutions in Task 3 and 4

A steady solution can be *stable* or *unstable*. One can envision a stable solution by the classical example of a bead sliding along the surface of a spherical bowl. At large time, the bead settles at the bottom of the bowl - a *stable* steady state - regardless of where it is initially located. If the bead is already located at the steady position, perturbing it away from that position will lead to a restoration back to the steady position. To understand an *unstable* steady solution, envision balancing a pencil on its tip. While this is a legitimate steady state, a small perturbation to the pencil at that state will lead to a large departure from the steady state (and no chance of coming back to it). Moreover, a pencil cannot spontaneously reach this unstable steady state from any initial position, other than the unstable steady state itself.