MAE502, Fall 2018 Homework #5

Hard copy of report is due 6:00 PM on the due date.

Task 0 (no point, but mandatory to complete for the report to be accepted)
Provide a statement to address whether collaboration occurred in completing this assignment. **This statement must be placed in the beginning of the first page of report.** See related clarifications in Homework #1.

You might find the following formulas useful:

$$\int_0^\infty \frac{\cos(bx)}{1+x^2} dx = \left(\frac{\pi}{2}\right) e^{-|b|}$$, where $|b|$ is the absolute value of b.

$$\int_0^\infty e^{-x} \cos(bx) dx = \frac{1}{1+b^2}$$

$$\int_0^\infty e^{-x^2} \cos(2bx) dx = \frac{\sqrt{\pi}}{2} e^{-b^2}$$

Task 1 (3 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, use the Fourier transform method to solve the PDE

$$\frac{\partial u}{\partial t} = (1 + 2t) \frac{\partial^2 u}{\partial x^2}$$

with the boundary condition,

$$u(x, 0) = e^{-x^2}$$

We expect a closed-form real solution without any unevaluated integral.

Task 2 (3 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, use the Fourier transform method to solve the PDE

$$\frac{\partial u}{\partial t} = 3t \frac{\partial u}{\partial x} - 2u$$

with the boundary condition,

$$u(x, 0) = e^{-x^2}$$

We expect a closed-form real solution without any unevaluated integral.

Task 3 (3 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, consider the PDE

$$\frac{\partial u}{\partial t} = (1 - t) \frac{\partial^2 u}{\partial x^2}$$

with the boundary condition,

$$u(x, 0) = \frac{1}{1+x^2}$$

Evaluate $u(x,t)$ at $x = 3$, $t = 2$. (Note: The key deliverable of this task is the exact value of $u(3,2)$. You may or may not need to find the full solution, $u(x, t)$ for all x and t, to answer the key question.)