MAE/MSE 502, Spring 2018 Homework #1 solution

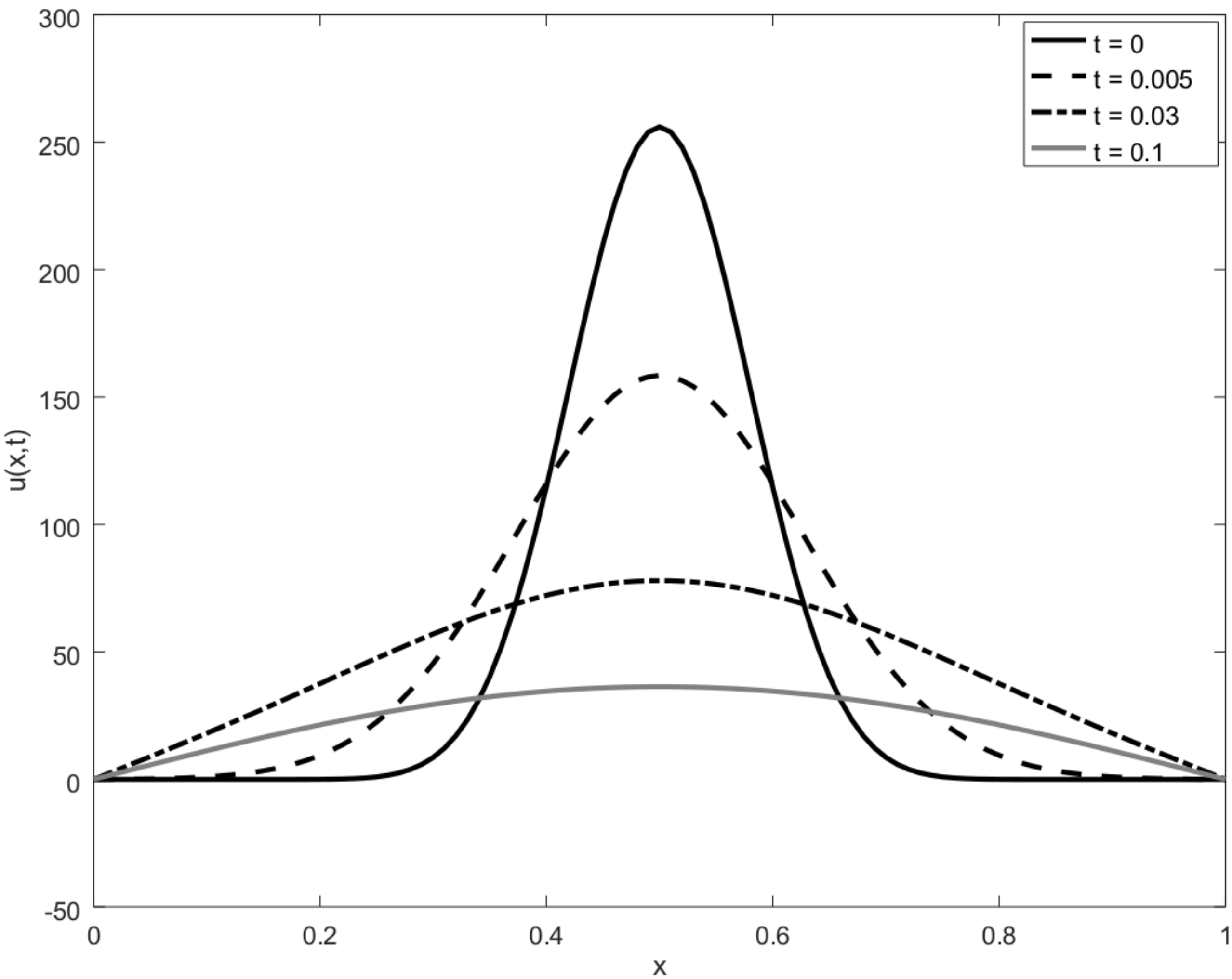
Task 1

(a)

$$u(x,t) = \sum_{n=1}^{\infty} a_n \sin(n\pi x)\, e^{-(n\pi)^2 t} \quad ,$$

$$a_n = 2\int_0^1 [1-\cos(2\pi x)]^8 \sin(n\pi x)\, dx \quad , n = 1, 2, 3, \ldots$$

Plot:

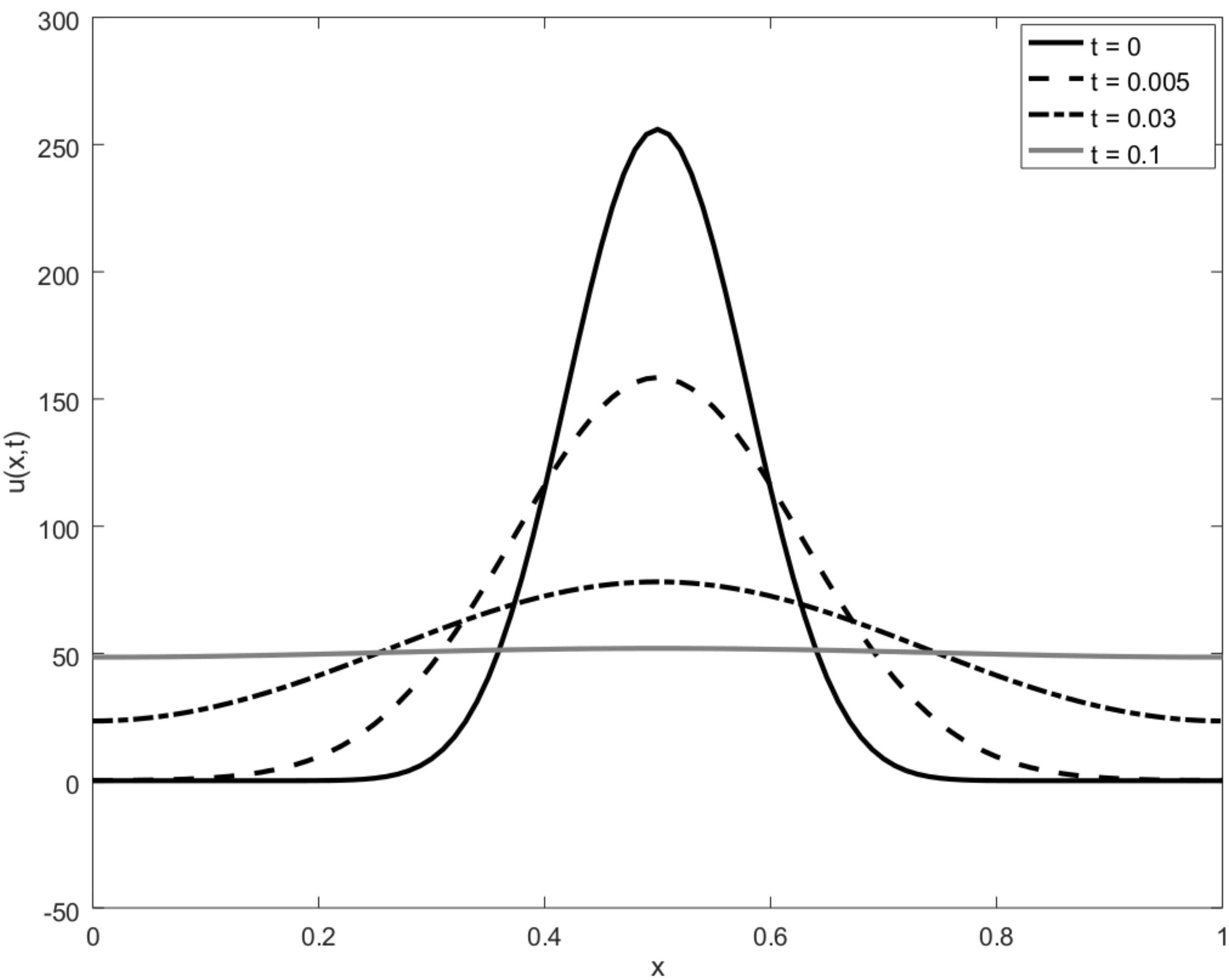


(b)

$$u(x,t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\pi x)\, e^{-(n\pi)^2 t} \quad ,$$

$$a_0 = \int_0^1 [1-\cos(2\pi x)]^8\, dx \quad , \quad a_n = 2\int_0^1 [1-\cos(2\pi x)]^8 \cos(n\pi x)\, dx \quad , n = 1, 2, 3, \ldots$$

Plot:

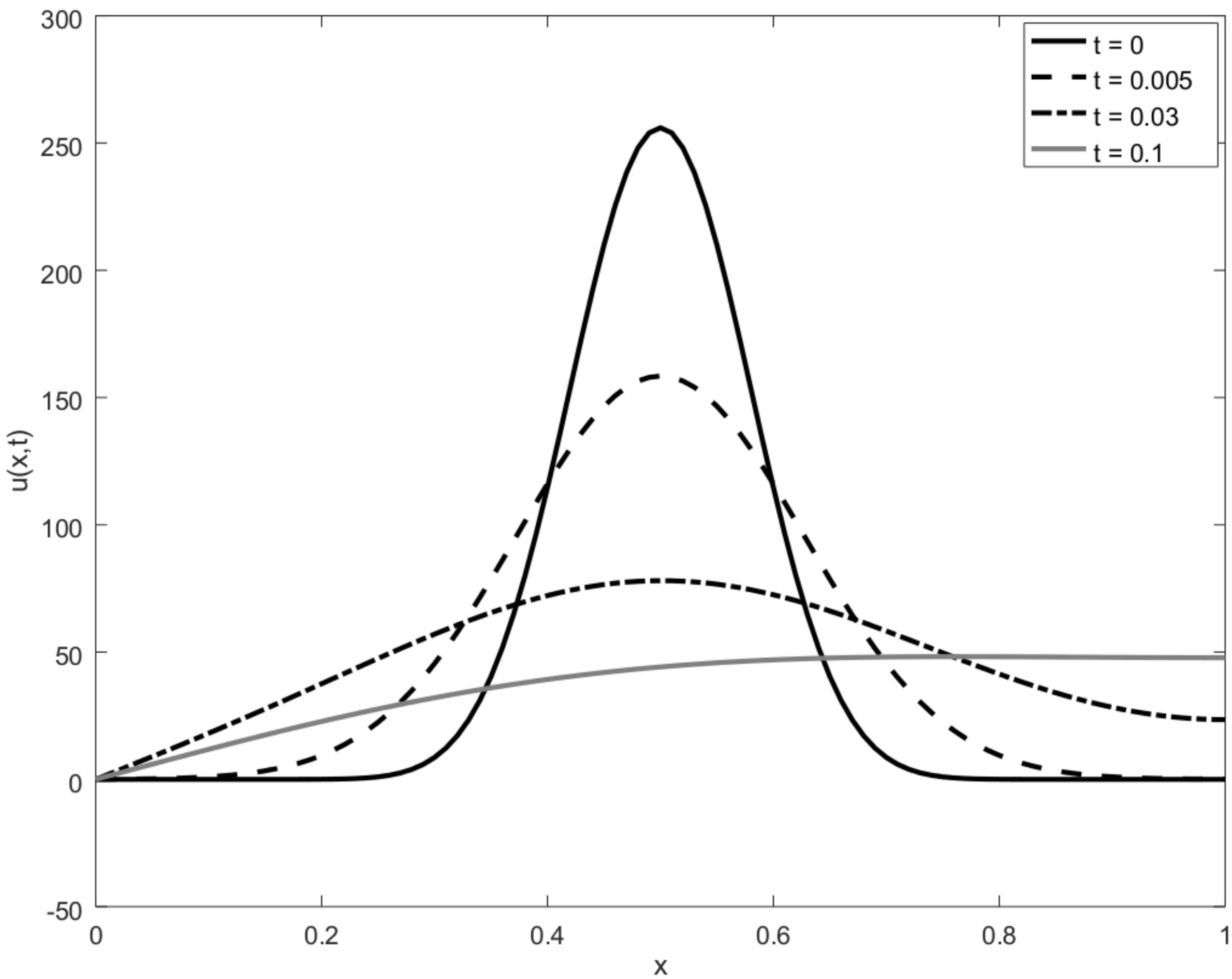


(c)

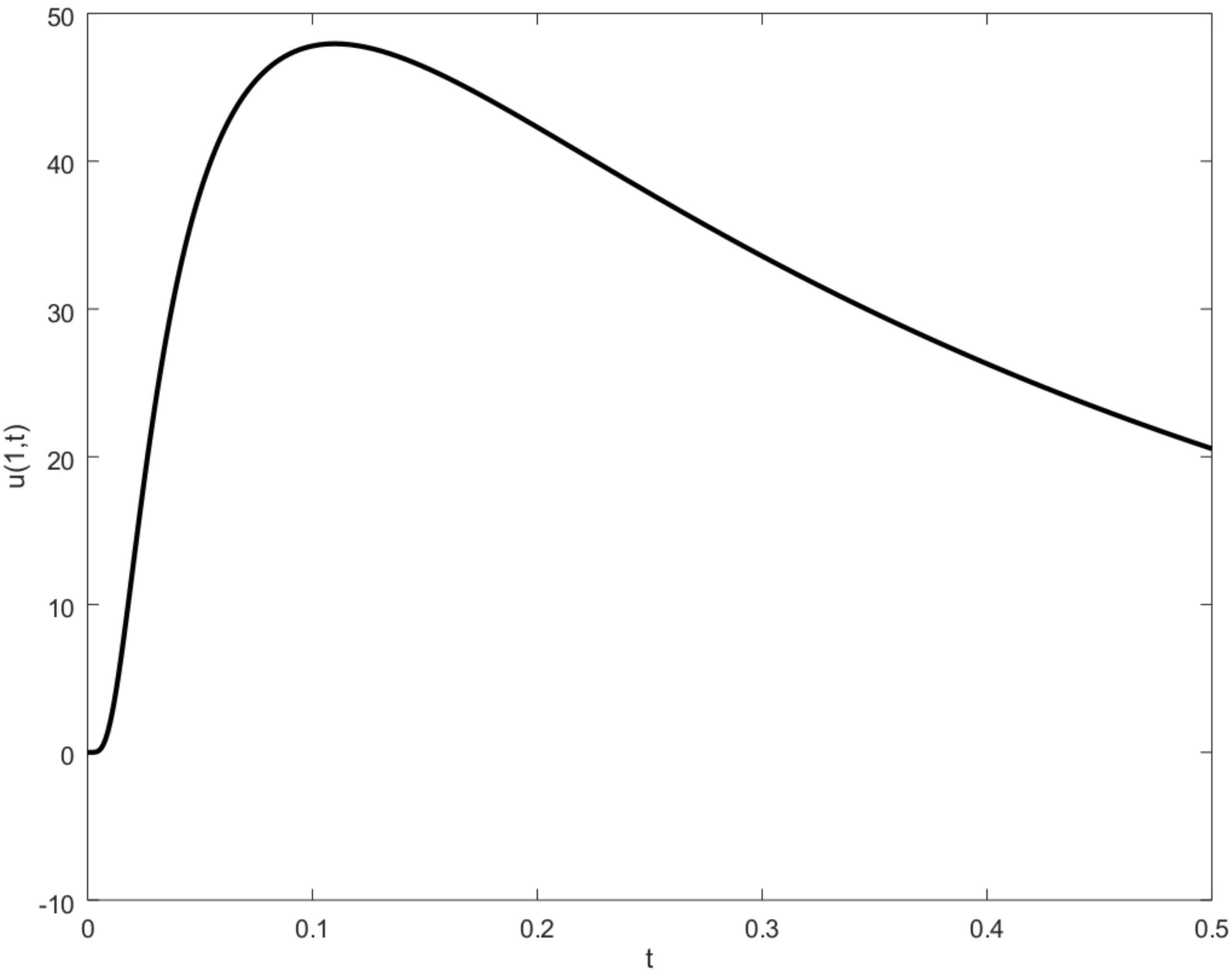
$$u(x,t) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{2}\right) e^{-\left(\frac{n\pi}{2}\right)^2 t}$$, where the summation is over <u>odd values</u> of *n* only.

$$a_n = 2\int_0^1 [1-\cos(2\pi x)]^8 \sin\left(\frac{n\pi x}{2}\right) dx \quad , n = 1, 3, 5, \ldots$$

Plot of u(x,t):



Plot of u(1,t) as a function of t:



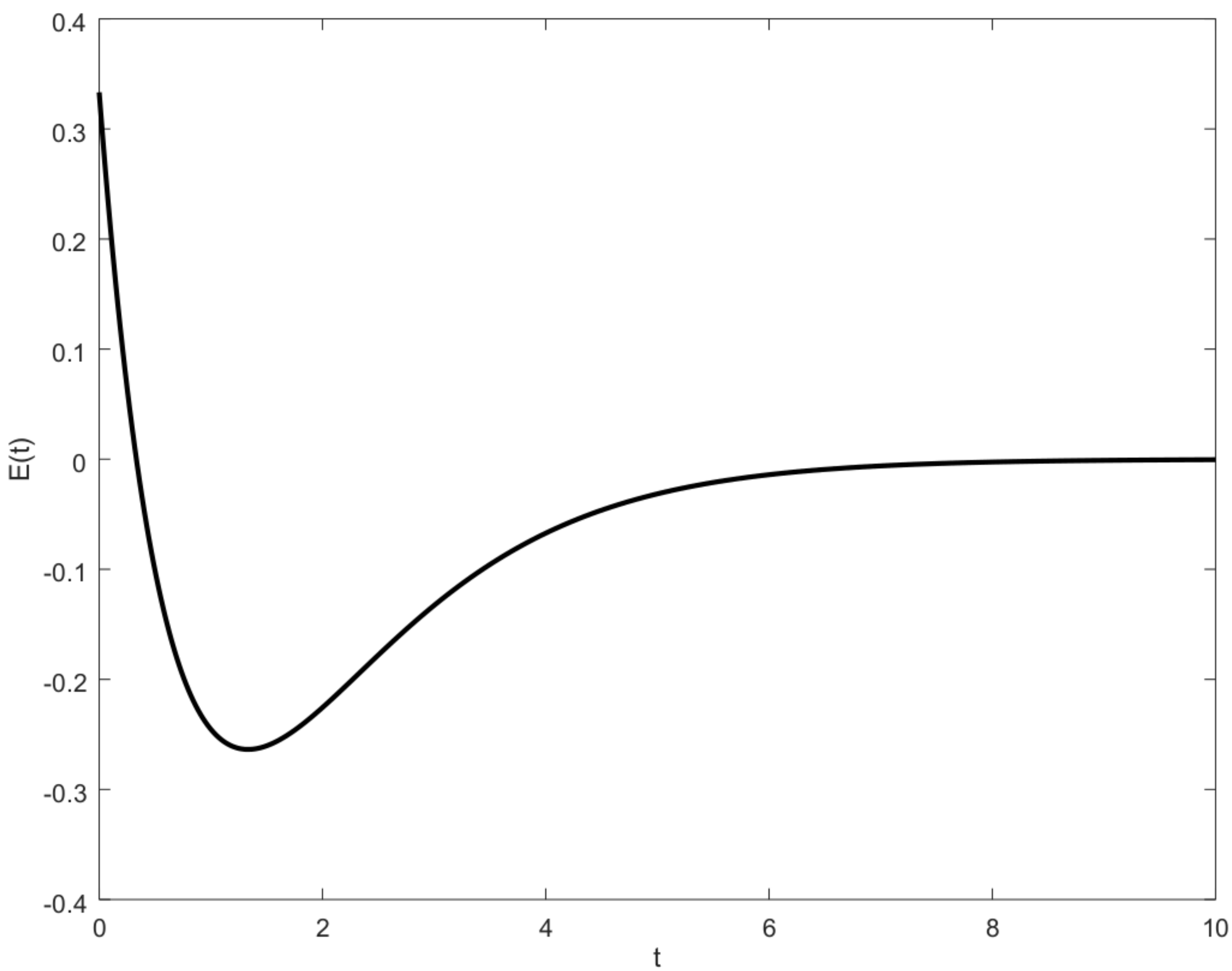
Task 2

$$u(x,t) = \sin(\pi x)\,(1+t)^{1-\pi^2}\,e^{-t} + \sin(3\pi x)\,(1+t)^{1-9\pi^2}\,e^{-t}$$

Task 3

$$E(t) = \left(\frac{1}{3} - t\right) e^{-t}$$

Plot of E(t):



Task 4

The solution will discussed in class.