MAE/MSE 502, Spring 2018 Homework #2 solution

Task 1

(a) & **(b)** All values of c along the real line, $-\infty < c < \infty$, are eigenvalues. The only exceptions are the isolated points at $c = -(n\pi/10)^2$, $n = 1, 3, 5, 7, ...$, where a solution for the eigenvalue problem does not exist.

The eigenfunctions are

$$G_c(x) = \frac{3\sinh\left(\sqrt{c}\ x\right)}{\sqrt{c}\ \cosh\left(5\ \sqrt{c}\right)} \quad \text{, if } c > 0$$

$$G_c(x) = 3\ x \quad \text{, if } c = 0$$

$$G_c(x) = \frac{3\sin\left(\sqrt{-c}\ x\right)}{\sqrt{-c}\ \cos\left(5\ \sqrt{-c}\right)} \quad \text{, if } c < 0 \text{ and } c \neq -(n\pi/10)^2,\ n = 1, 3, 5, 7, ...$$

The plot for (b) is in the next page.

(c) The orthogonality relation does not hold.

(d) No.

Task 2

$$u(x,t) = 3\ \cosh(\pi t) + \frac{4}{\pi}\sinh(\pi t) + t\ \cos(\pi x) + \cos\left(\sqrt{3}\ \pi t\right)\cos(2\pi x)$$

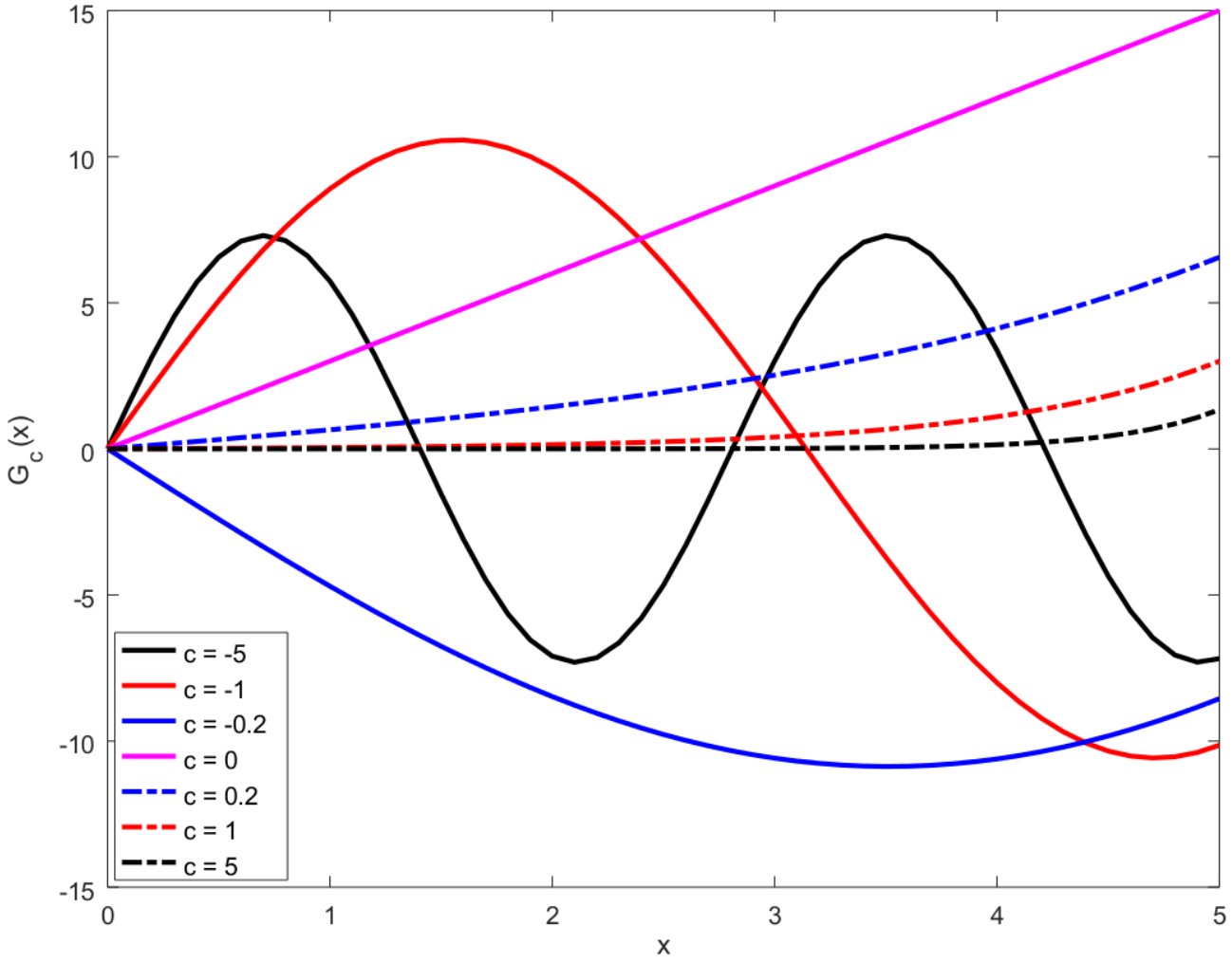
Task 3

(a) The system has infinitely many solutions.

(b) $u(x,y) = a_0 + \dfrac{\cos(2\pi y)\cosh(2\pi x)}{2\pi\sinh(2\pi)}$, where a_0 is an arbitrary constant.

(c) The contour plot is in the last page.

Plot for Task 1b:



Plot for Task 3c: