MAE/MSE 502, Spring 2018, Homework #3

Hard copy of report is due 6:00 PM on the due date. Computer codes used to complete the tasks should be included in the report.

Task 0 (no point, but mandatory to complete for the report to be accepted)
Provide a statement to address whether collaboration occurred in completing this assignment. **This statement must be placed in the beginning of the first page of report.** If no collaboration occurred, simply state "No collaboration". This implies that the person submitting the report has not helped anyone or received help from anyone on this assignment. If collaboration occurred, provide the name of collaborator (only one allowed), a list of the task(s) on which collaboration occurred, and descriptions of the extent of collaboration. Please see related clarifications in Homework #1.

Task 1 (4 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, consider the following four PDEs:

$$\text{(I)}\quad \frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} \qquad \text{(II)}\quad \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \qquad \text{(III)}\quad \frac{\partial u}{\partial t} = \frac{\partial^3 u}{\partial x^3} \qquad \text{(IV)}\quad \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \;.$$

Solve each PDE with periodic boundary conditions in the x-direction (i.e., $u(0, t) = u(2\pi, t)$, $u_x(0, t) = u_x(2\pi, t)$, and so on) and the boundary condition in t-direction given as

$$u(x,0) = \left[\frac{1-\cos(x)}{2}\right]^{40} \quad .$$ (Note the 40th power in the given function.)

For Eq. (IV), an additional boundary condition in t-direction is given as

$$u_t(x, 0) = 0 \;.$$

You may express the solution as an infinite (Fourier) series. For each case, plot the solution as a function of x at specific values of t, given as following:

(I) Plot $u(x, t)$ at $t = 0$, 1, 2, and π. Collect all 4 curves in one plot.
(II) Plot $u(x, t)$ at $t = 0$, 0.03, 0.15, and 3. Collect all 4 curves in one plot.
(III) Plot $u(x, t)$ at $t = 0$ and 0.005. Collect the two curves in one plot.
(IV) Plot $u(x, t)$ at $t = 0$, 0.4, and 1.6. Collect all 3 curves in one plot.

[See the general remark below HW1-Task1 on estimating the appropriate number of terms to keep in the Fourier series. For this task, a significant deduction will be assessed on an inaccurate plot of solution due to keeping too few terms in the Fourier series.]

Task 2 (2 points)
For $u(x, t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, solve the PDE,

$$\frac{\partial u}{\partial t} = 4\,\frac{\partial^3 u}{\partial x^3} - \frac{\partial^5 u}{\partial x^5} - u \ ,$$

with periodic boundary conditions in the x-direction and the boundary condition in the t-direction given as

$$u(x, 0) = 5 + \sin(x) + \cos(2x) \ .$$

We expect a closed-form analytic solution which consists of only a finite number of terms and no unevaluated integrals. The solution should be expressed in real numbers and functions. A deduction will be assessed on any imaginary number "i" ($= \sqrt{-1}$) that is left in the solution.

Task 3 (2 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, solve the PDE,

$$\frac{\partial^3 u}{\partial t^3} = \frac{\partial^4 u}{\partial x^4} - \frac{\partial^8 u}{\partial x^8} \quad ,$$

with periodic boundary conditions in the x-direction, and the boundary condition in the t-direction given as

(i) $u(x, 0) = \sin(x)$ (ii) $u_t(x, 0) = 2 + \cos(x)$ (iii) $u_{tt}(x, 0) = 3$.

We expect a closed-form anlytic solution which consists of only a finite number of terms and without any unevaluated integrals. The solution should be expressed in real numbers and functions. A deduction will be assessed on any imaginary number "i" ($= \sqrt{-1}$) that is left in the solution.

Task 4 (1 point)
For $G(x)$ defined over the interval of $1 \le x \le 2$, consider the boundary value problem with the ODE,

$$x^2\, G'' + 2\, x\, G' = A\, x^2 \quad \text{(where } G' \text{ is } dG/dx\text{)} ,$$

and boundary conditions

$$G'(1) = 1\ , \quad G'(2) = 3\ .$$

(a) For what value of A will a solution (or solutions) exist for this system? **(b)** Under the special value of A obtained from Part (a), will the solution of the system be unique? Explain why. (No credit for Part (b) without a proper explanation.)
[Hint: You may or may not need to solve $G(x)$ to answer the key questions.]