MAE/MSE 502, Spring 2018   Homework #4 and #5 solutions

HW4

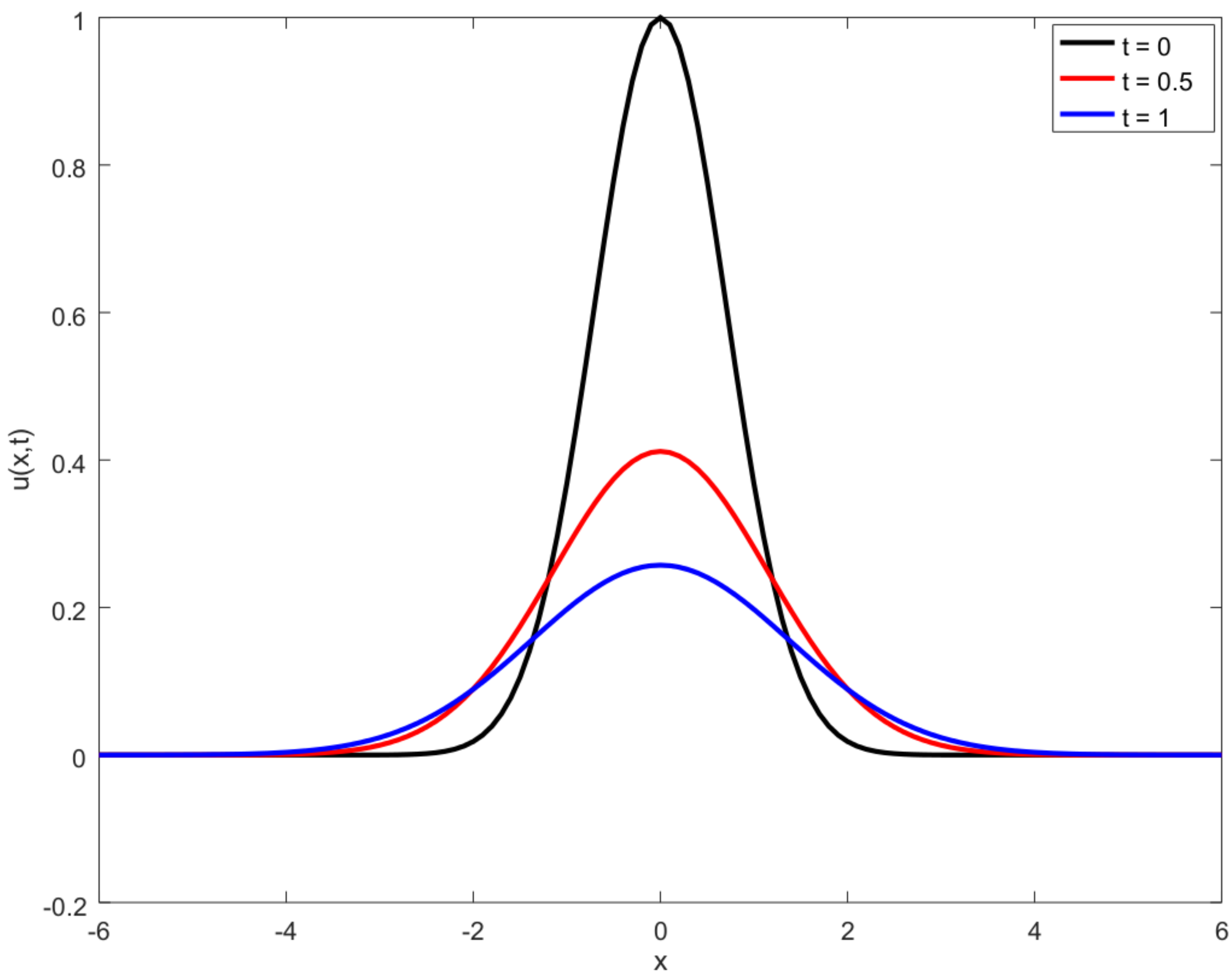
Task 1

$$u(x,t) = \frac{1}{\sqrt{\pi}} \int_0^\infty e^{-\frac{\omega^2}{4}} (1+t)^{-(\omega^2+1)} \cos(\omega x)\, d\omega \ .$$

It is not required, but the solution can actually be expressed in closed form as

$$u(x,t) = \frac{1}{1+t} \frac{e^{\left[\frac{-x^2}{1+4\ln(1+t)}\right]}}{\sqrt{1+4\ln(1+t)}} \ .$$

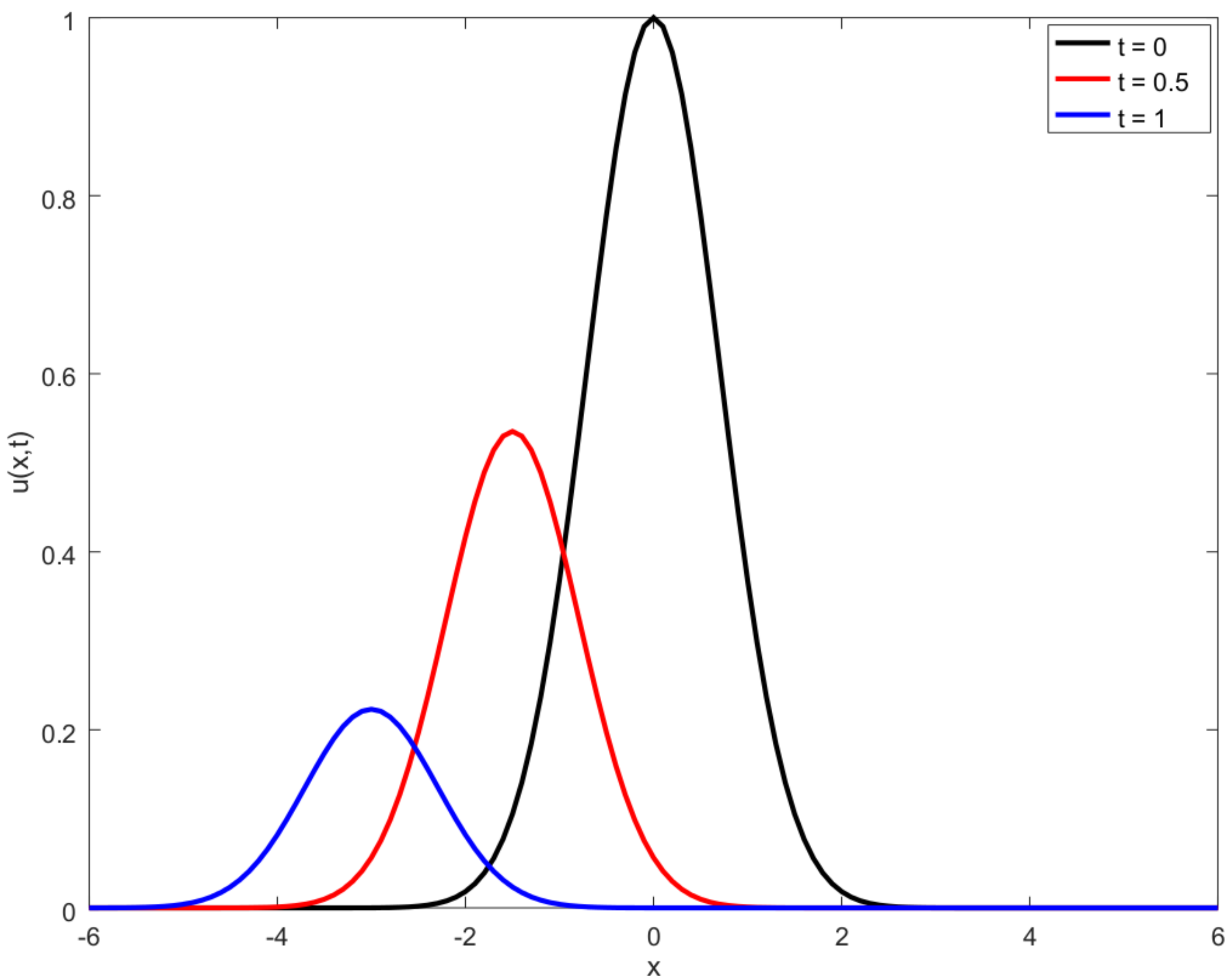
Plot:



Task 2

$$u(x,t) = e^{-[t+\frac{t^2}{2}+(x+3t)^2]}$$

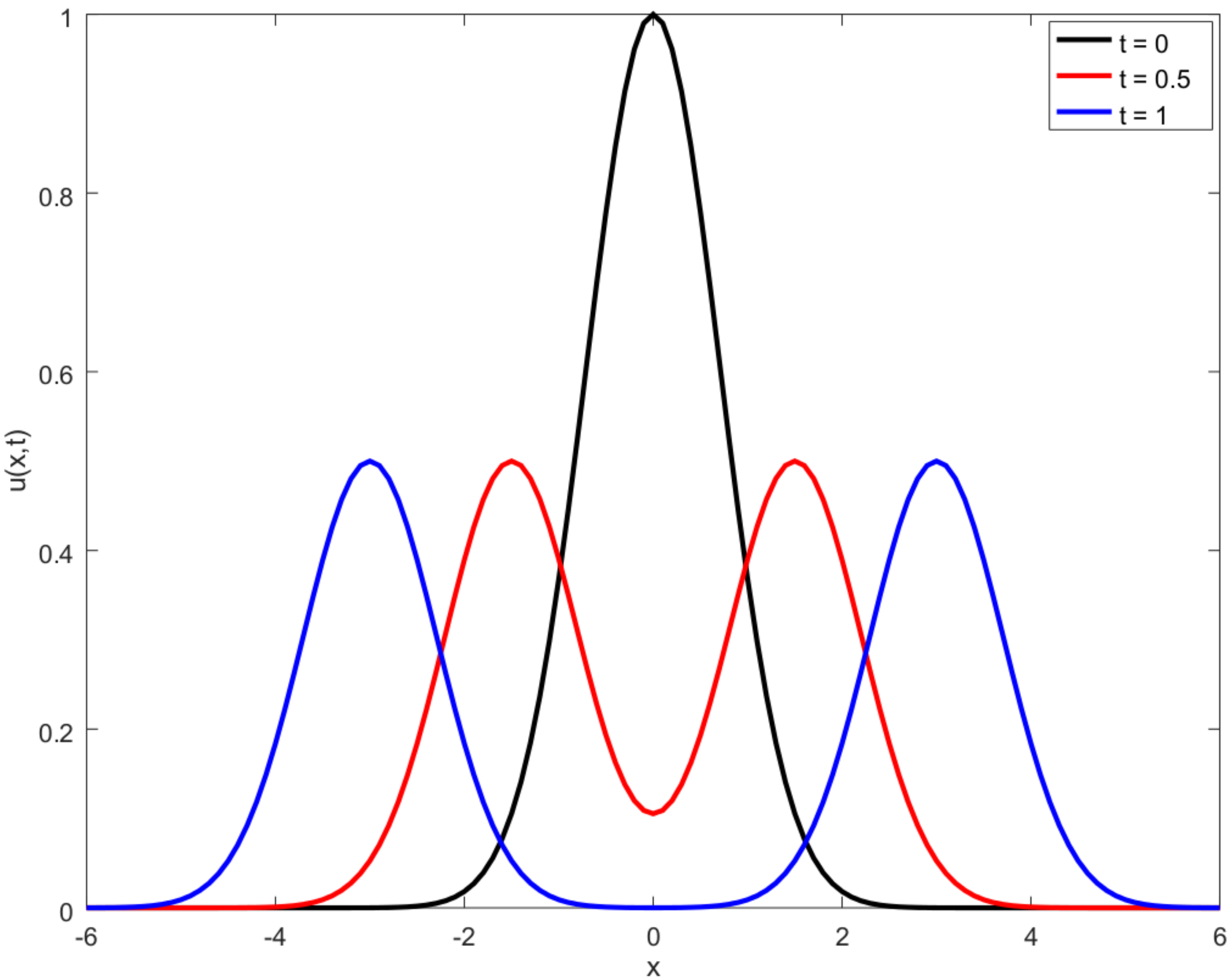
Plot:



Task 3

$$u(x,t) = \frac{1}{2}\left[e^{-(x+3t)^2} + e^{-(x-3t)^2}\right]$$

Plot:



The solutions for Task 2 and Task 3 have been discussed in Lecture 23 and 25, respectively, using the Method of Characteristics. As announced in class, in HW4 these two problems must be solved using the Fourier transform method to receive full credit.

HW5

Task 1

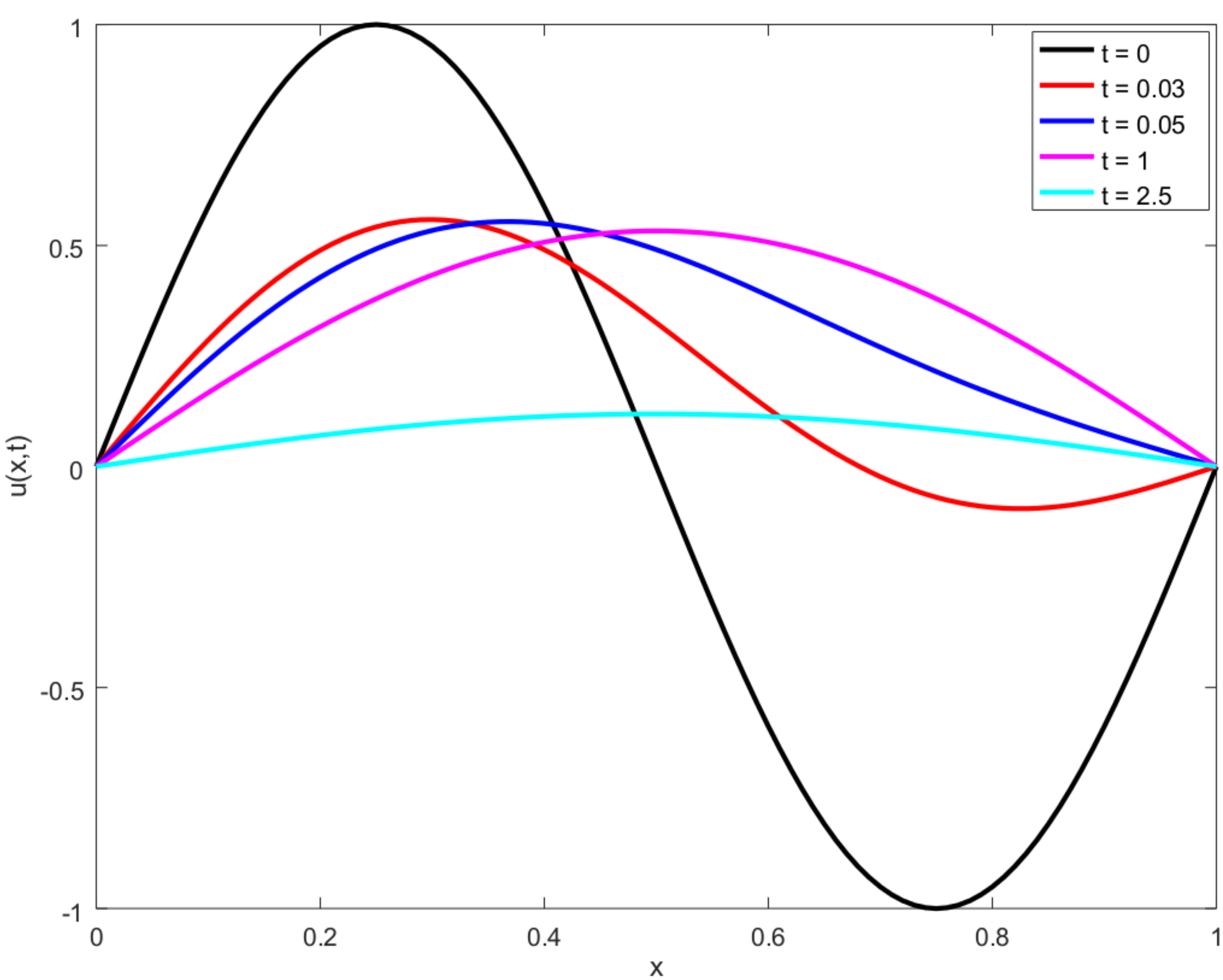
$u(x,t) = \sum_{n=1}^{\infty} a_n(t)\, \sin(n\pi x)$, where

$a_n(t) = a_n(0)\, e^{-(n\pi)^2 t} + \hat{q}_n \left[\frac{e^{-t} - e^{-(n\pi)^2 t}}{(n\pi)^2 - 1}\right]$, where $a_2(0) = 1$ and $a_n(0) = 0$ for all $n \neq 2$,

and

$\hat{q}_n = 2\int_0^1 (x - x^2)\, \sin(n\pi x)\, dx$.

Plot:



Task 2

$$u(x,t) = \ln(1+t) + \frac{x^2}{2(1+t)}$$

Task 3

$$u(x,t) = 2-2e^{-t}-t + [1-\cos(t)]\cos(x) + \cos(t)\sin(x)$$