MAE/MSE 502, Spring 2018, Homework #4

Hard copy of report is due 6:00 PM on the due date. Computer codes used to complete the tasks should be included in the report.

You might find the following formula useful for this homework:

$$\int_0^\infty e^{-x^2} \cos(2bx)\, dx = \frac{\sqrt{\pi}}{2} e^{-b^2} \qquad \textbf{Eq. (1)}$$

Task 0 (no point, but mandatory to complete for the report to be accepted)
Provide a statement to address whether collaboration occurred in completing this assignment. **This statement must be placed in the beginning of the first page of report.** If no collaboration occurred, simply state "No collaboration". This implies that the person submitting the report has not helped anyone or received help from anyone on this assignment. If collaboration occurred, provide the name of collaborator (only one allowed), a list of the task(s) on which collaboration occurred, and descriptions of the extent of collaboration. Please see related clarifications in Homework #1.

Task 1 (3 points)
For $u(x,t)$ defined on the infinite domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE,

$$(1+t)\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - u$$

with the boundary condition,

$$u(x, 0) = \exp(-x^2) .$$

You may express the solution as an integral. Plot the solution $u(x, t)$ as a function of x at $t = 0$, 0.5, and 1.

Task 2 (2 points)
For $u(x,t)$ defined on the infinite domain of $-\infty < x < \infty$ and $t \geq 0$, use the Fourier transform method to solve the PDE,

$$\frac{\partial u}{\partial t} = 3\frac{\partial u}{\partial x} - (1+t)\, u \quad ,$$

with the boundary condition,

$$u(x, 0) = \exp(-x^2) .$$

For this task, we expect a closed form analytic solution without any unevaluated integral. Plot the solution $u(x, t)$ as a function of x at $t = 0$, 0.5, and 1.

Task 3 (2 points)
For $u(x,t)$ defined on the infinite domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE,

$$\frac{\partial^2 u}{\partial t^2} = 9\,\frac{\partial^2 u}{\partial x^2} \quad ,$$

with the boundary conditions,

(i) $u(x, 0) = \exp(-x^2)$

(ii) $u_t(x, 0) = 0$

For this task, we expect a closed form analytic solution without any unevaluated integral. Plot the solution $u(x, t)$ as a function of x at $t = 0$, 0.5, and 1.

Hint: For Task 2 and 3, the integral formula in Eq. (1) can be used twice to process both Fourier transform and inverse Fourier transform. For Task 1, it can be used to process Fourier transform while numerical integration is needed to complete the inverse transform. For Task 3, you might find the trigonometric identity useful: cos(A)cos(B) = cos(A+B)/2 + cos(A–B)/2.