MAE/MSE 502, Spring 2018, Homework #5

Hard copy of report is due 6:00 PM on the due date. Computer codes used to complete the tasks should be included in the report.

Task 0 (no point, but mandatory to complete for the report to be accepted)
Provide a statement to address whether collaboration occurred in completing this assignment. **This statement must be placed in the beginning of the first page of report.** If no collaboration occurred, simply state "No collaboration". This implies that the person submitting the report has not helped anyone or received help from anyone on this assignment. If collaboration occurred, provide the name of collaborator (only one allowed), a list of the task(s) on which collaboration occurred, and descriptions of the extent of collaboration. Please see related clarifications in Homework #1.

Task 1 (3 points)
For $u(x,t)$ defined on the domain of $0 \leq x \leq 1$ and $t \geq 0$, solve the PDE,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 50\,(x - x^2)\,e^{-t} \quad ,$$

with the boundary conditions,

(i) $u(0, t) = 0$, (ii) $u(1, t) = 0$, (iii) $u(x, 0) = \sin(2\pi\, x)$.

You may express the solution as an infinite series. Plot the solution $u(x, t)$ as a function of x at $t = 0$, 0.03, 0.05, 1, and 2.5. Please collect all 5 curves in a single plot and clearly label the curves.

Task 2 (3 points)
For $u(x,t)$ defined on the domain of $0 \leq x \leq 1$ and $t \geq 0$, solve the PDE,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - \frac{1}{2}\left(\frac{x}{1+t}\right)^2 \quad ,$$

with the boundary conditions,

(i) $u_x(0, t) = 0$, (ii) $u_x(1, t) = \dfrac{1}{1+t}$, (iii) $u(x, 0) = \dfrac{x^2}{2}$.

(Note that the first two boundary conditions are imposed on the derivative of u.)

We expect a closed-form analytic solution with only a finite number of terms and without any unevaluated integral.

Task 3 (3 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, solve the PDE,

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \cos(x) - 2\,\mathrm{e}^{-t} \quad ,$$

with periodic boundary conditions in the x-direction (i.e. $u(0, t) = u(2\pi, t)$, $u_x(0, t) = u_x(2\pi, t)$, and so on), and the boundary conditions in t-direction given as

(i) $u(x, 0) = \sin(x)$, (ii) $u_t(x, 0) = 1$.

We expect a closed-form analytic solution with only a finite number of terms and without any unevaluated integral.