

MAE/MSE 502, Spring 2018, Homework #6

Hard copy of report is due 6:00 PM on the due date. Computer codes used to complete the tasks should be included in the report.

Task 0 (no point, but mandatory to complete for the report to be accepted)

Provide a statement to address whether collaboration occurred in completing this assignment.

This statement must be placed in the beginning of the first page of report. If no collaboration occurred, simply state "No collaboration". This implies that the person submitting the report has not helped anyone or received help from anyone on this assignment. If collaboration occurred, provide the name of collaborator (only one allowed), a list of the task(s) on which collaboration occurred, and descriptions of the extent of collaboration. Please see related clarifications in Homework #1.

Task 1 (3 points)

For $u(x,t)$ defined on the infinite domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE,

$$\frac{\partial u}{\partial t} + (1+u) \frac{\partial u}{\partial x} = -0.5 u$$

with the boundary condition,

$$u(x,0) = P(x),$$

where

$$\begin{aligned} P(x) &= 0, \text{ if } x < 0 \\ &= x^2, \text{ if } 0 \leq x \leq 1 \\ &= 1, \text{ if } x > 1. \end{aligned}$$

Plot the solution $u(x, t)$ as a function of x at $t = 0, 0.8$, and 1.6 . The recommended range for plotting is $-2 \leq x \leq 6$.

Task 2 (2 points)

For $u(x,t)$ defined on the infinite domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE,

$$\frac{\partial u}{\partial t} + t x \frac{\partial u}{\partial x} = t x u, \quad ,$$

with the boundary condition,

$$u(x, 0) = \exp(-x^2).$$

Plot the solution $u(x, t)$ as a function of x at $t = 0, 0.8$, and 1 . The recommended range for plotting is $-6 \leq x \leq 6$.

Task 3 (2 points)

For $u(x,t)$ defined on the infinite domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE,

$$\frac{\partial u}{\partial t} + (x+t-0.5u) \frac{\partial u}{\partial x} = 2x + 2t - u \quad ,$$

with the boundary condition,

$$u(x, 0) = 2 \quad .$$

Plot the solution $u(x, t)$ as a function of x at $t = 0, 0.1, 0.3,$ and 0.5 . The recommended range for plotting is $-2 \leq x \leq 2$.