

## MAE502, Spring 2019 Homework # 2

Hard copy of report is due 6:00 PM on the due date. The report should include a statement on collaboration, and computer code(s) used for the assignment. See the cover page of Homework #1 for the rules on collaboration.

### Prob 1 (2.5 points)

For  $u(x,y)$  defined on the square domain of  $0 \leq x \leq 1$  and  $0 \leq y \leq 1$ , solve the PDE

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 ,$$

with the boundary conditions,

$$(i) u(0, y) = 0 \quad (ii) u(1, y) = \sin(\pi\sqrt{y}) \quad (iii) u(x, 0) = 0 \quad (iv) u(x, 1) = 2(x - x^4) .$$

Make a contour plot of the solution,  $u(x, y)$ . See additional note in the last page for an example of using Matlab to make a contour plot. The recommended contour interval for the plot is 0.1. For this problem, we expect the solution to be expressed as an infinite series. Please see the remark below HW1-Prob1 on how to truncate the series and numerically compute the expansion coefficients.

### Prob 2 (1.5 points)

For  $u(x,y)$  defined on the rectangular domain of  $0 \leq x \leq 4$  and  $0 \leq y \leq 1$ , consider the PDE

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 ,$$

with the boundary conditions ( $u_x$  and  $u_y$  denote  $\partial u/\partial x$  and  $\partial u/\partial y$ , respectively),

$$(i) u_x(0, y) = 0 \quad (ii) u_x(4, y) = 0 \quad (iii) u_y(x, 0) = 1 + 2\cos(\pi x) \quad (iv) u_y(x, 1) = 1$$

(a) Test the solvability condition on the system. Based on it, which of the following is true?

(I) The system has no solution. (II) The system has a unique solution. (III) The system has multiple solutions. If your answer is (I), no need to proceed further. Otherwise, proceed to Part (b).

(b) Find the solution(s) of the system. If your answer for Part (a) is (III), please clearly write out what those multiple solutions are.

### Prob 3 (2.5 points)

For  $u(x,y)$  defined on the rectangular domain of  $1 \leq x \leq 2$  and  $0 \leq y \leq \pi$ , solve the PDE

$$x^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - 2x \frac{\partial u}{\partial x} + 5u = 0 ,$$

with the boundary conditions ( $u_x$  and  $u_y$  denote  $\partial u/\partial x$  and  $\partial u/\partial y$ , respectively),

$$(i) u_x(1, y) = 6 \cos(3y) \quad (ii) u(2, y) = 6 \cos(3y) \quad (iii) u_y(x, 0) = 0 \quad (iv) u_y(x, \pi) = 0$$

For this problem, we expect a closed-form analytic solution that consists of only a finite number of terms and without any unevaluated integral. A deduction will be assessed on the solution otherwise.

**Prob 4** (2.5 points)

For  $u(x,y)$  defined on the square domain of  $0 \leq x \leq \pi$  and  $0 \leq y \leq \pi$ , solve the PDE

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + 16u = 0,$$

with the boundary conditions ( $u_x$  and  $u_y$  denote  $\partial u/\partial x$  and  $\partial u/\partial y$ , respectively),

$$(i) u(0, y) = 3 \quad (ii) u_x(\pi, y) = \cos(4y) + \cos(5y) \quad (iii) u_y(x, 0) = 0 \quad (iv) u_y(x, \pi) = 0$$

For this problem, we expect a closed-form analytic solution that consists of only a finite number of terms and without any unevaluated integral. A deduction will be assessed on the solution otherwise.

**Additional Note: Using Matlab to make a contour plot**

The following Matlab code makes a contour plot for  $u(x,y) = \sin(2\pi x)\exp(-2y)$  over the domain of  $0 \leq x \leq 1$  and  $0 \leq y \leq 1$ , using the contour levels of  $(-0.9, -0.7, -0.5, -0.3, -0.1, -0.05, 0.05, 0.1, 0.3, 0.5, 0.7, 0.9)$ . The contours for  $u = -0.7, -0.3, 0.3,$  and  $0.7$  are labeled. It is essential to provide the coordinates of the grid (x2d and y2d in this example) as the input for the contour function. Without this piece of information, Matlab would not know the grid spacing and the correct directions of  $x$  and  $y$ . A black-and-white contour plot is acceptable as long as the contours are properly labeled.

```
clear
x = [0:0.01:1]; y = [0:0.01:1];
for i = 1:length(x)
    for j = 1:length(y)
        u(i,j) = sin(2*pi*x(i))*exp(-2*y(j));
        x2d(i,j) = x(i); y2d(i,j) = y(j);
    end
end
[C,h] = contour(x2d,y2d,u,[-0.9:0.2:-0.1 -0.05 0.05 0.1:0.2:0.9]);
clabel(C,h,[-0.7 -0.3 0.3 0.7]);xlabel('x');ylabel('y')
```

