

MAE 502, Spring 2019 HW2 Solution

Prob 1

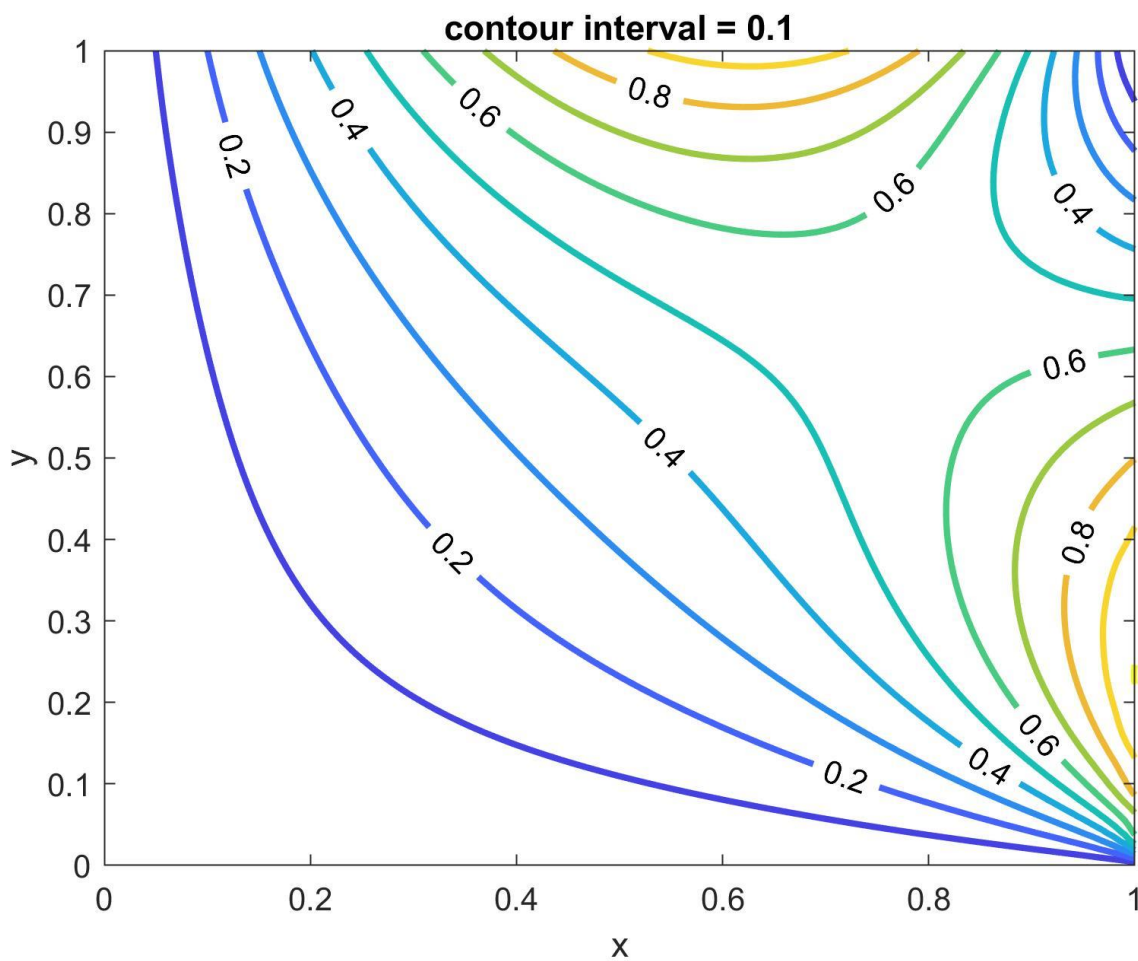
$$u(x, y) = \sum_{n=1}^{\infty} a_n \sin(n\pi y) \sinh(n\pi x) + b_n \sin(n\pi x) \sinh(n\pi y)$$

where

$$a_n = \frac{2}{\sinh(n\pi)} \int_0^1 \sin(\pi\sqrt{y}) \sin(n\pi y) dy$$

$$b_n = \frac{2}{\sinh(n\pi)} \int_0^1 2(x - x^4) \sin(n\pi x) dx$$

Plot:



Prob 2

(a) There are infinitely many solutions

(b)

$$u(x, y) = C + y + \frac{2}{\pi} [\sinh(\pi y) - \coth(\pi) \cosh(\pi y)] \cos(\pi x)$$

where C is an arbitrary constant (which corroborates the conclusion from Part (a)).

Prob 3

$$u(x, y) = \left(\frac{x^4}{2} - \frac{4}{x} \right) \cos(3y)$$

Prob 4

$$u(x, y) = 3 \cos(4x) + x \cos(4y) + \frac{1}{3 \cosh(3\pi)} \sinh(3x) \cos(5y)$$