MAE502, Spring 2019 Homework # 3

Hard copy of report is due 6:00 PM on the due date. The report should include a statement on collaboration, and computer code(s) used for the assignment. See the cover page of Homework #1 for the rules on collaboration.

Prob 1 (1 point)
Consider the eigenvalue problem for $G(x)$ (with c as the eigenvalue) defined on the interval, $0 \le x \le 1$,

$$\frac{d^2 G}{dx^2} = c\, G$$

with the boundary conditions,

(i) $G(0) = 0.5G(1)$, (ii) $G'(0) = 0$ (Note that the second b.c. is imposed on the derivative of G.)

(a) Is the system given above a Sturm-Liouville system?
(b) Find all eigenvalue(s) and the corresponding eigenfunction(s) for the system.

Prob 2 (2 points)
(a) For $u(x, t)$ defined on the domain of $0 \le x \le 1$ and $t \ge 0$, solve the Wave equation,

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

with the boundary conditions:

(i) $u(0, t) = 0$ (ii) $u(1, t) = 0$ (iii) $u(x, 0) = \left[\sin(\sqrt{x} - x)\right]^2$ (iv) $u_t(x, 0) = 0$.

Plot the solution $u(x, t)$ as a function of x at $t = 0, 0.25, 0.5, 0.75$, and 1.0. Please collect all 5 curves in one plot.
(b) Repeat (a) but with the second boundary condition changed to: (ii) $u_x(1, t) = 0$. Plot the solution $u(x, t)$ as a function of x at $t = 0, 0.25, 0.5, 0.75$, and 1.0.

Prob 3 (3 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, consider the PDE (in which U and K are constants)

$$\frac{\partial u}{\partial t} = U\frac{\partial u}{\partial x} + K\frac{\partial^2 u}{\partial x^2}$$

with periodic boundary conditions in the x-direction (i.e., $u(0, t) = u(2\pi, t)$, $u_x(0, t) = u_x(2\pi, t)$, and so on), and the boundary condition in the t-direction given as

$$u(x, 0) = \frac{[1-\cos(x)]^{12}}{4096} + \frac{[1-\cos\left(x-\frac{\pi}{2}\right)]^{12}}{2048} .$$

Solve the PDE by Fourier series expansion. Plot the solution $u(x, t)$ at $t = 1.2$ for the three cases with
(i) $U = 2, K = 0$, (ii) $U = 0, K = 0.05$, and (iii) $U = 2, K = 0.05$. Also, plot the solution at $t = 0$ (which is the same for all three cases). Please collect all four curves in one plot.

Prob 4 (2 points)
For $u(x,t)$ defined on the domain of $0 \leq x \leq 2\pi$ and $t \geq 0$, solve the PDE,

$$\frac{\partial u}{\partial t} = (1+t)\frac{\partial^2 u}{\partial x^2} + t\,\frac{\partial^3 u}{\partial x^3} + t\,\frac{\partial^4 u}{\partial x^4} + u$$

with periodic boundary conditions in the x-direction, and the boundary condition in the t-direction given as

$u(x, 0) = 1 + \sin(x)$.

We expect a closed-form solution which consists of only a finite number of terms and without any unevaluated integrals. The solution should be real and should be expressed in real numbers and functions. A deduction will be assessed on the solution otherwise.

Prob 5 (2 points)
For $u(x,t)$ defined on the domain of $0 \leq x \leq 2\pi$ and $t \geq 0$, solve the PDE

$$\frac{\partial^2 u}{\partial t^2} = 4\,\frac{\partial^2 u}{\partial x^2} + \frac{\partial^4 u}{\partial x^4} + 3\,u$$

with periodic boundary conditions in the x-direction and the following boundary conditions in the t-direction,

(i) $u(x, 0) = \cos(2x)$,

(ii) $u_t(x, 0) = \sin(x)$.

We expect a closed-form solution which consists of only a finite number of terms and without any unevaluated integrals. The solution should be real and should be expressed in real numbers and functions. A deduction will be assessed on the solution otherwise.